METHOD AND APPARATUS FOR ADAPTIVE REAL-TIME SIGNAL CONDITIONING, PROCESSING, ANALYSIS, QUANTIFICATION, COMPARISON, AND CONTROL

This application claims the benefit of United States Provisional Patent Application No.60/416,562 filed on October 7, 2002, which is incorporated herein by reference in its

5   entirety.

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TECHNICAL FIELD

The present invention relates to methods, processes and apparatus for real-time measuring

15   and analysis of variables. In particular, it relates to adaptive real-time signal conditioning, processing, analysis, quantification, comparison, and control. This invention also relates to generic measurement systems and processes, that is, the proposed measuring arrangements are not specially adapted for any specific variables, or to one particular environment. This invention also relates to methods and corresponding apparatus for measuring which extend

20   to different applications and provide results other than instantaneous values of variables. The invention further relates to post-processing analysis of measured variables and to statistical analysis.

BACKGROUND ART

Due to the rapid development of digital technology since the 1950's, the development of analog devices has been essentially squeezed out to the periphery of data acquisition equipment only. It could be argued that the conversion to digital technology is justified by the flexibility,

5   universality, and low cost of modern integrated circuits. However, it usually comes at the price of high complexity of both hardware and software implementations. The added complexity of digital devices stems from the fact that all operations must be reduced to the elemental manipulation of binary quantities using primitive logic gates. Therefore, even such basic operations as integration and differentiation of functions require a very large number of such

10   gates and/or sequential processing of discrete numbers representing the function sampled at many points. The necessity to perform a very large number of elemental operations limits the ability of digital systems to operate in real time and often leads to substantial dead time in the instruments. On the other hand, the same operations can be performed instantly in an analog device by passing the signal representing the function through a simple RC circuit.

15   Further, all digital operations require external power input, while many operations in analog devices can be performed by passive components. Thus analog devices usually consume much less energy, and are more suitable for operation in autonomous conditions, such as mobile communication, space missions, prosthetic devices, etc.

It is widely recognized (see, for example, Paul and Hüper (1993)) that the main obstacle to

20   robust and efficient analog systems often lies in the lack of appropriate analog definitions and the absence of differential equations corresponding to known digital operations. When proper definitions and differential equations are available, analog devices routinely outperform corresponding digital systems, especially in nonlinear signal processing (Paul and Hüper, 1993). However, there are many signal processing tasks for which digital algorithms are well known,

25   but corresponding analog operations are hard to reproduce. One example, which is widely recognized to fall within this category, is related to the use of signal processing techniques

based on *order statistics*[1]. Order statistic (or rank) filters are gaining wide recognition for their ability to provide robust estimates of signal properties and are becoming the filters of choice for applications ranging from epileptic seizure detection (Osorio et al., 1998) to image processing (Kim and Yaroslavsky, 1986). However, since such filters work by sorting, or order-
5   ing, a set of measurements their implementation has been constrained to the digital domain. As pointed out by some authors (Paul and Hüper, 1993, for example), the major problem in analog rank processing is the lack of an appropriate differential equation for 'analog sorting'. There have been several attempts to implement such sorting and to build continuous-time rank filters without using delay lines and/or clock circuits. Examples of these efforts include optical
10   rank filters (Ochoa et al., 1987), analog sorting networks (Paul and Hüper, 1993; Opris, 1996), and analog rank selectors based on minimization of a non-linear objective function (Urahama and Nagao, 1995). However, the term 'analog' is often perceived as only 'continuous- time', and thus these efforts fall short of considering the threshold continuity, which is necessary for a truly analog representation of differential sorting operators. Even though Ferreira (2000, 2001)
15   extensively discusses threshold distributions, these distributions are only piecewise-continuous and thus do not allow straightforward introduction of differential operations with respect to threshold.

Nevertheless, fuelled by the need for robust filters that can operate in real time and on a low energy budget, analog implementation of traditionally digital operations has recently gained
20   in popularity aided by the rapid progress in analog Very Large Scale Integration (VLSI) technology (Mead, 1989; Murthy and Swamy, 1992; Kinget and Steyaert, 1997; Lee and Jen, 1993). However, current efforts to implement digital signal processing methods in analog devices still employ an essentially digital philosophy. That is, a continuous signal is passed through a delay line which samples the signal at discrete time intervals. Then the individual samples
25   are processed by a cascade of analog devices that mimic elemental digital operations (Vlassis et al., 2000). Such an approach fails to exploit the main strength of analog processing, which is the ability to perform complex operations in a single step without employing the 'divide and conquer' paradigm of the digital approach.

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[1]See, for example, Arnold et al. (1992) for the definitions and theory of order statistics.

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Perhaps the most common digital waveform device is the analog-to-digital converter (ADC). Among the salient characteristics of ADCs are their sampling frequency, measurement resolution, power dissipation, and system complexity. Sampling frequency is typically dictated by the signal of interest and/or the requirements of the application. As the frequency

5 content of the signal of interest increases and the sampling frequency increases, resolution decreases both in terms of the absolute number of bits available in an ADC and in terms of the effective number of bits (ENOB), or accuracy, of the measurement. Power needs typically increase with increasing sampling frequency. The system complexity is increased if continuous monitoring of an input signal is required (real-time operation). As an example, high-end

10 oscilloscopes can capture fast transient events, but are limited by record length (the number of samples that can be acquired) and dead time (the time required to process, store, or display the samples and then reset for more data acquisition). These limitations affect any data acquisition system in that, as the sampling frequency increases, resources will ultimately be limited at some point in the processing chain. In addition, the higher the acquisition speed,

15 the more negative effects such as clock crosstalk, jitter, and synchronization issues combine to reduce system performance.

It is highly desirable to extract signal characteristics or preprocess data prior to digitization so that the requirements on the ADCs are reduced and higher quality data can be obtained. In the past, one common technique was to use an analog memory to sample a fast signal and

20 then the analog memory would be clocked out at a low speed and digitized with a moderately high resolution ADC. While this technique works, it suffers from significant degradation due to clock feedthrough, non-linear effects of the analog memories chosen, and limited record length. Another technique used is to multiplex a high-speed signal to a number of lower speed but higher resolution ADCs using an interleaved clock. Again, the technique works but never

25 realizes the best performance of a single channel due to the high clock noise and inevitable differences in processing channels.

The introduction of the Analysis of VAriables Through Analog Representation (AVATAR) methodology (see Nikitin and Davidchack (2003a) and US Patent Application 09/921,524, which are incorporated herein by reference in their entirety) is aimed to address many aspects

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of modern data acquisition and signal processing tasks by offering solutions that combine the benefits of both digital and analog technology, without the drawbacks of high cost, high complexity, high power consumption, and low reliability. The AVATAR methodology is based on the development of a new mathematical formalism, which takes into consideration the

5   limited precision and inertial properties of real physical measurement systems. Using this formalism, many problems of signal analysis can be expressed in a content-sentient form suitable for analog implementation. Specific devices for a wide variety of signal processing tasks can be built from a few universal processing units. Thus, unlike traditional analog solutions, AVATAR offers a highly modular approach to system design. Most practical applications of

10   AVATAR, however, are far from obvious, and their development requires technical solutions unavailable in the prior art. For example, AVATAR introduces the definitions of analog filters and selectors. Nonetheless, the practical implementations of these filters offered by AVATAR are often unstable and suffer from either lack of accuracy or lack of convergence speed, and thus are unsuitable for real-time processing of nonstationary signals. Another

15   limitation of AVATAR lies in the definition of the *threshold filter*. Namely, a threshold filter in AVATAR depends *only* on the difference between the displacement and the input variables, and is expressed as a scalar function of *only* the displacement variable, which limits the scope of applicability of AVATAR. As another example, the analog counting in AVATAR is introduced through modulated density, and thus the instantaneous counting rate is expressed

20   as a product of a rectified time derivative of the signal and the output of a probe. Even though this definition theoretically allows counting without dead time effect, its practical implementations are cumbersome and inefficient.

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DISCLOSURE OF INVENTION

BRIEF SUMMARY OF THE INVENTION

The present invention, collectively designated as Adaptive Real-Time Embodiments for Multivariate Investigation of Signals (ARTEMIS), overcomes the shortcomings of the prior art

5   by directly processing the data in real-time in the analog domain, identifying the events of interest so that continuous digitization and digital processing is not required, performing direct, noise-resistant measurements of the salient signal characteristics, and outputting a signal proportional to these characteristics that can be digitized without the need for highspeed front-end sampling.

10   In the face of the overwhelming popularity of digital technology, simple analog designs are often overlooked. Yet they often provide much cheaper, faster, and more efficient solutions in applications ranging from mobile communication and medical instrumentation to counting detectors for high-energy physics and space missions. The current invention, collectively designated as Adaptive Real-Time Embodiments for Multivariate Investigation of

15   Signals (ARTEMIS), explores a new mathematical formalism for conducting adaptive content-sentient real-time signal processing, analysis, quantification, comparison, and control, and for detection, quantification, and prediction of changes in signals. The method proposed herein overcomes the limitations of the prior art by directly processing the data in real-time in the analog domain, identifying the events of interest so that continuous digitization and digital

20   processing is not required, performing a direct measurement of the salient signal characteristics such as energy and arrival rate, and outputting a signal proportional to these characteristics that can be digitized without the need for highspeed front-end sampling. In addition, the analog systems can operate without clocks, which reduces the noise introduced into the data.

A simplified diagram illustrating multimodal analog real-time signal processing is shown in figure 1. The process comprises the step of Threshold Domain Filtering in combination with at least one of the following steps: (a) Multimodal Pulse Shaping, (b) Analog Rank Filtering, and (c) Analog Counting.

5   Threshold Domain Filtering is used for separation of the features of interest in a signal from the rest of the signal. In terms of a threshold domain, a 'feature of interest' is either a point inside of the domain, or a point on the boundary of the domain. A typical Threshold Domain Filter can be composed of (asynchronous) comparators and switches, where the comparators operate on the differences between the components of the incoming variable(s) and the

10   corresponding components of the control variable(s). For example, for the domain defined as a product of two *ideal comparators* represented by the Heaviside unit step function $\theta(x)$, $\mathcal{D} = \theta[x(t) - D]\ \theta[\dot{x}(t)]$ (with two control levels, D and zero), a point *inside* (that is, $\mathcal{D} = 1$) corresponds to a positive-slope signal of the magnitude greater than D, and the stationary points of $x(t)$ above the threshold D can be associated with the points on the boundary of

15   this domain.

Multimodal Pulse Shaping can be used for embedding the incoming signal into a threshold space and thus enabling extraction of the features of interest by the Threshold Domain Filtering. A typical Multimodal Pulse Shaper transforms at least one component of the incoming signal into at least two components such that one of these two components is a (partial)

20   derivative of the other. For example, for identification of the signal features associated with the stationary points of a signal $x(t)$, the Multimodal Pulse Shaping is used to output both the signal $x(t)$ and its time derivative $\dot{x}(t)$.

Analog Rank Filtering can be used for establishing and maintaining the analog control levels of the Threshold Domain Filtering. It ensures the adaptivity of the Threshold Domain

25   Filtering to changes in the measurement conditions, and thus the optimal separation of the features of interest from the rest of the signal. For example, the threshold level D in the domain $\mathcal{D} = \theta[x(t) - D]\ \theta[\dot{x}(t)]$ can be established by means of Analog Rank Filtering to separate the stationary points of interest from those caused by noise. Note that the Analog Rank Filtering outputs the control levels indicative of the salient properties of the input signal(s),

and thus can be used as a stand-alone embodiment of ARTEMIS for adaptive real-time signal conditioning, processing, analysis, quantification, comparison, and control, and for detection, quantification, and prediction of changes in signals.

Analog Counting can be used for identification and quantification of the crossings of the threshold domain boundaries, and its output(s) can be either the instantaneous rate(s) of these crossings, or the rate(s) in moving window of time. A typical Analog Counter consists of a time-differentiator followed by a rectifier, and an optional time-integrator.

Further scope of the applicability of the invention will be clarified through the detailed description given hereinafter. It should be understood, however, that the specific examples, while indicating preferred embodiments of the invention, are presented for illustration only. Various changes and modifications within the spirit and scope of the invention should become apparent to those skilled in the art from this detailed description. Furthermore, all the mathematical expressions and the examples of hardware implementations are used only as a descriptive language to convey the inventive ideas clearly, and are not limitative of the claimed invention.

BRIEF DESCRIPTION OF FIGURES

Figure 1. Simplified diagram of a multimodal analog system for real-time signal processing.

Figure 2. Representative step (a) and impulse (b) responses of a continuous comparator.

5   **Figure 3**. Defining output $D_q(t)$ of a rank filter as a level curve of the distribution function $\Phi(D, t)$.

Figure 4. Amplitude and counting densities computed for the fragment of a signal shown at the top of the figure.

Figure 5. Comparison of the rates measured with the boxcar (thin solid line) and the

10   'triple-integrator' test function ($n = 3$ in equation (14), thick line) with $\tau = T/6$. The respective test functions are shown in the upper left corner of the figure.

Figure 6. Illustration of bimodal pulse shaping used for the amplitude and timing measurements of a short-duration event.

Figure 7. A simplified principal schematic of a signal processing system for a two-detector

15   particle telescope.

Figure 8. Example of analog measurement of instantaneous rate of signal's extrema.

Figure 9. Example of analog counting of coincident maxima.

Figure 10. Coincident counting in a time-of-flight window: (a). 'Ideal' instrument; (b) 'Realistic' instrument.

20   **Figure 11**. Principle schematic of implementation of an adaptive real-time rank filter given by equation (24).

Figure 12. Simplified diagram of an Adaptive Analog Rank Filter (AARF).

Figure 13. Principle schematic of a 3-comparator implementation of AARF.

Figure 14. Principle schematic of a delayed comparator.

25   **Figure 15**. Principle schematic of an averaging comparator.

Figure 16. Comparison of the performance of the analog rank filter given by equation (24) to that of the 'exact' quantile filter in a boxcar moving window of width T.

Figure 17. Example of using an internal reference (baseline) for separating signal from noise.

Figure 18. Principle schematic of establishing baseline by means of quartile (trimean) filter.

Figure 19. Principle schematic of implementation of an adaptive real-time rank selector given by equation (28).

5   **Figure 20.** Simplified diagram of an Adaptive Analog Rank Selector (AARS).

Figure 21. Removing static and dynamic impulse noise from a monochrome image by AARSs.

Figure 22. Comparison of quartile outputs ($q = 1/4$, $1/2$, and $3/4$) of ARF and EARL for amplitudes and counts.

10 **Figure 23.** Simplified principle schematic of a Bimodal Analog Sensor Interface System (BASIS).

Figure 24. Simplified principle schematic of an Integrated Output Module of BASIS.

Figure 25. Simulated example of the performance of BASIS used with a PMT.

Figure 26. Modification of BASIS used for detection of onsets/offsets of light pulses.

15 **Figure 27.** Principle schematic of a monoenergetic Poisson pulse generator.

Figure 28 Simulated performance of a monoenergetic Poisson pulse generator.

TERMS AND DEFINITIONS

WITH ILLUSTRATIVE EXAMPLES

For convenience, the essential terms used in the subsequent detailed description of the invention are provided below, along with their definitions adopted for the purpose of this disclosure. Some examples clarifying and illustrating the meaning of the definitions are also

5   provided. Note that the sections and equations in this part are numbered separately from the rest of the disclosure. Additional explanatory information on relevant terms and definitions can be found in US Patent Application 09/921,524 and US Provisional Patent Application No.60/416,562, which are incorporated herein by reference in their entirety. Some other terms and their definitions which might appear in this disclosure will be provided in the detailed

10   description of the invention.

D-1 Continuous comparators and probes

Consider a simple measurement process whereby a signal $x(t)$ is compared to a threshold value D. The ideal measuring device would return '0' or '1' depending on whether $x(t)$ is

15   larger or smaller than D. The output of such a device is represented by the Heaviside unit step function $\theta\left[D - x(t)\right]$, which is discontinuous at zero. However, the finite precision of real measurements inevitably introduces uncertainty in the output whenever $x(t) \approx D$. To describe this property of a real measuring device, we represent its output by a *continuous* function $\mathcal{F}_{\Delta D}\left[D - x(t)\right]$, where the *width parameter* ΔD characterizes the threshold interval over which

20   the function changes from '0' to '1' and, therefore, reflects the measurement precision level. We call $\mathcal{F}_{\Delta D}(D)$ the threshold *step* response of a *continuous comparator*. Because of the continuity of this function, its derivative $f_{\Delta D}(D) = \mathrm{d}\mathcal{F}_{\Delta D}/\mathrm{d}D$ exists everywhere, and we call it the comparator's threshold *impulse* response, or a *probe* (Nikitin et al., 2003; Nikitin and Davidchack, 2003a,b). This *threshold continuity* of the output of a comparator is the

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key to a truly analog representation of such a measurement. Examples of step and impulse responses of a continuous comparator are shown in figure 2. We further assume, for simplicity, that the probe is a unimodal even function, that is, $f_{\Delta D}(D)$ has only a single maximum and $f_{\Delta D}(D) = f_{\Delta D}(-D)$.

5   In practice, many different circuits can serve as comparators, since any continuous monotonic function with constant unequal horizontal asymptotes will produce the desired response under appropriate scaling and reflection. It may be simpler to implement a comparator described by an odd function $\widetilde{\mathcal{F}}_{\Delta D}$ which relates to the response $\mathcal{F}_{\Delta D}$ as

$$\widetilde{\mathcal{F}}_{\Delta D} = A\left(2\mathcal{F}_{\Delta D} - 1\right), \tag{D-1}$$

10   where A is an arbitrary (nonzero) constant. For example, the voltage-current characteristic of a subthreshold transconductance amplifier (Mead, 1989; Urahama and Nagao, 1995) can be described by the hyperbolic tangent function, $\widetilde{\mathcal{F}}_{\Delta D} = A\tanh(D/\Delta D)$, and thus such an amplifier can serve as a continuous comparator. For specificity, this response function is used in the numerical examples of this disclosure. A practical implementation of the probe $f_{\Delta D}$

15   corresponding to the comparator $\mathcal{F}_{\Delta D}$ can be conveniently accomplished as a finite difference

$$f_{\Delta D}(D) \approx \frac{\delta\widetilde{\mathcal{F}}_{\Delta D}(D)}{4A\,\delta D} = \frac{1}{4A\,\delta D}\left[\widetilde{\mathcal{F}}_{\Delta D}(D + \delta D) - \widetilde{\mathcal{F}}_{\Delta D}(D - \delta D)\right], \tag{D-2}$$

where δD is a relatively small fraction of ΔD.

Note that the terms 'comparator' and 'discriminator' might be used synonymously in this disclosure.

D-2   ANALOG RANK FILTERS (ARFs)

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Consider the measuring process in which the difference between the *threshold variable* D and the scalar signal $x(t)$ is passed through a comparator $\mathcal{F}_{\Delta D}$, followed by a linear time averaging filter with a continuous impulse response $w(t)$. The output of this system can be written as

$$\Phi(D, t) = w(t) * \mathcal{F}_{\Delta D}\left[D - x(t)\right], \tag{D-3}$$

25   where the asterisk denotes convolution. The physical interpretation of the function $\Phi(D, t)$ is

the (time dependent) *cumulative distribution function* of the signal $x(t)$ in the moving time window $w(t)$ (Nikitin and Davidchack, 2003b). In the limit of high resolution (small ΔD), equation (D-3) describes the 'ideal' distribution (Ferreira, 2001). Notice that $\Phi(D, t)$ is viewed as a function of *two* variables, *threshold* D and time t, and is continuous in both variables.

5   The output of a *quantile* filter of order q in the moving time window $w(t)$ is then given by the function $D_q(t)$ defined implicitly as

$$\Phi[D_q(t), t] = q, \qquad 0 < q < 1 \,. \tag{D-4}$$

Viewing the function $\Phi(D, t)$ as a surface in the three-dimensional space (t, D, Φ), we immediately have a geometric interpretation of $D_q(t)$ as that of a level (or contour) curve obtained

10   from the intersection of the surface $\Phi = \Phi(D, t)$ with the plane $\Phi = q$, as shown in figure 3. Based on this geometric interpretation, one can develop various explicit as well as feedback representations for analog rank filters, including such generalizations as L filters and α-trimmed mean filters (Nikitin and Davidchack, 2003b).

Note that the terms analog 'rank', 'quantile', 'percentile', and 'order statistic' filters are

15   often synonymous and might be used alternatively in this disclosure.

D-2.1   Quantile filters for modulated densities

Let us point out (see Nikitin and Davidchack, 2003a,b, for example) that various threshold densities can be viewed as different appearances of a general *modulated threshold den-*

20   *sity* (MTD)

$$\phi(D, t) = \frac{w(t) * \{K(t) f_{\Delta D}[D - x(t)]\}}{w(t) * K(t)} \,, \tag{D-5}$$

where $K(t)$ is a unipolar *modulating signal*. Various choices of the modulating signal allow us to introduce different types of threshold densities and impose different conditions on these densities. For example, the simple *amplitude density* is given by the choice $K(t) = $ const,

25   and setting $K(t)$ equal to $|\dot{x}(t)|$ leads to the *counting density*. The significance of the defini-

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tion of the time dependent counting (threshold crossing) density arises from the fact that it characterizes the *rate of change* in the analyzed signal, which is one of the most important characteristics of a dynamic system. Notice that the amplitude density is proportional to the time the signal spends in a vicinity of a certain threshold, while the counting density is proportional to the number of 'visits' to this vicinity by the signal. Figure 4 shows both the amplitude and counting densities computed for the fragment of a signal shown in the top panel of the figure. Note that the amplitude density has a sharp peak at every signal extremum, while the counting density has a much more regular shape.

An expression for the quantile filter for a modulated density can be written as

$$\frac{\mathrm{d}D_q}{\mathrm{d}t} = \frac{w_T(t) * \{K(t)\,(q - \mathcal{F}_{\Delta D}\,[D_q - x(t)])\}}{\tau\,w(t) * \{K(t)\,f_{\Delta D}\,[D_q - x(t)]\}} \; , \tag{D-6}$$

and the physical interpretation of such a filter depends on the nature of the modulating signal. For example, a median filter in a rectangular moving window for $K(t) = |\ddot{x}(t)|\,f_{\Delta D \dot{x}}\,[\dot{x}(t)]$ yields $D_{1/2}(t)$ such that half of the extrema of the signal $x(t)$ in the window are below this threshold.

SELECTED ACRONYMS AND WHERE THEY FIRST APPEAR

Selected Notations and Where They First Appear

DETAILED DESCRIPTION OF THE INVENTION

The Detailed Description of the Invention is organized as follows.

Section 1 (p. 17) provides the definition of the threshold domain function and Threshold Domain Filtering, and explains its usage for feature extraction.

5   Section 2 (p. 19) deals with quantification of crossings of threshold domain boundaries by means of Analog Counting.

Section 3 (p. 22) introduces Multimodal Pulse Shaping as a way of embedding an incoming signal into a threshold space and thus enabling extraction of the features of interest by the Threshold Domain Filtering. Subsection 3.1 describes Analog Bimodal Coincidence (ABC) 10   counting systems as an example of a real-time signal processing utilizing Threshold Domain Filtering in combination with Analog Counting and Multimodal Pulse Shaping.

Section 4 (p. 26) presents various embodiments of Analog Rank Filters which can be used in ARTEMIS in order to reconcile the conflicting requirements of the robustness and adaptability of the control levels of the Threshold Domain Filtering. Subsection 4.1 describes the Adaptive 15   Analog Rank Filters (AARFs) and Adaptive Analog Rank Selectors (AARSs), while §4.2 introduces the Explicit Analog Rank Locators (EARLs). Subsection 4.3 describes the Bimodal Analog Sensor Interface System (BASIS) as an example of an analog signal processing module operatable as a combination of Threshold Domain Filtering, Analog Counting, and Analog Rank Filtering.

20   As an additional illustration of ARTEMIS, §5 (p. 46) describes a technique and a circuit for generation of monoenergetic Poissonian pulse trains with adjustable rate and amplitude through a combination of Threshold Domain Filtering and Analog Counting.

1 Threshold domain filtering

Threshold Domain Filtering is used for separation of the features of interest in a signal from the rest of the signal. In terms of a threshold domain, a 'feature of interest' is either a point inside of the domain, or a point on the boundary of the domain. In an electrical apparatus, e.g., a typical Threshold Domain Filter can be composed of (asynchronous) comparators and switches, where the comparators operate on the differences between the components of the incoming variable(s) and the corresponding components of the control variable(s). For example, for the domain defined as a product of two *ideal comparators* represented by the Heaviside unit step function $\theta(x)$, $\mathcal{D} = \theta[x(t) - D]\,\theta[\dot{x}(t)]$ (with two control levels, D and zero), a point *inside* (that is, $\mathcal{D} = 1$) corresponds to a positive-slope signal of the magnitude greater than D, and the stationary points of $x(t)$ above the threshold D can be associated with the points on the boundary of this domain. More generally, as used in the present invention, a Threshold Domain Filter is defined by its mathematical properties regardless of their physical implementation.

Defining threshold domain   Let us assume that a *continuous* signal $\mathbf{y} = \mathbf{y}(\mathbf{a}, t)$ depends on some spatial coordinates $\mathbf{a}$ and time t. Thus, in a vicinity of $(\mathbf{a}, t)$, this signal can be characterized by its value $\mathbf{y}(\mathbf{a}, t)$ at this point along with its partial derivatives $\partial \mathbf{y}(\mathbf{a}, t)/\partial a_i$ and $\partial \mathbf{y}(\mathbf{a}, t)/\partial t$ at this point. These values (of the signal and its derivatives) can be viewed as coordinates of a point $\mathbf{x} = \mathbf{x}(\mathbf{a}, t)$ in a *threshold space*, where the vector $\mathbf{x}$ consists of the signal $\mathbf{y}$ and its various partial derivatives. A particular *feature* of interest can thus be defined as a certain region in the threshold space as follows.

Let us describe an 'ideal' *threshold domain* by a (two-level) function $\mathcal{D}(\mathbf{D}, \mathbf{x})$ such that

$$
\begin{cases}
\mathcal{D}(\mathbf{D}, \mathbf{x}) = q_1 & \text{if } \mathbf{x} \text{ is inside domain} \\
\mathcal{D}(\mathbf{D}, \mathbf{x}) = q_2 & \text{otherwise}
\end{cases}
, \tag{1}
$$

where $\mathbf{D}$ is a vector of the *control levels* of the threshold filter. Without loss of generality, we can set $q_1 = 1$ and $q_2 = 0$. For example, in a physical space, an ideal *cuboid* with the edge lengths a, b, and c, centered at (x_0, y_0, z_0), can be represented by

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$$\mathcal{D}(x, y, z) = \theta \left[1 - 4 \left(\frac{x - x_0}{a} \right)^2 \right] \theta \left[1 - 4 \left(\frac{y - y_0}{b} \right)^2 \right] \theta \left[1 - 4 \left(\frac{z - z_0}{c} \right)^2 \right], \qquad (2)$$

where we have assumed constant control levels and thus $\mathcal{D}$ is a function of x, y, and z only. Note that for a 'real', or 'fuzzy', domain the transition from q_2 to q_1 happens monotonically over some finite interval (layer) of a characteristic thickness $\Delta\sigma$. The transition to a 'real'

5 threshold domain can be accomplished, for example, by replacing the ideal comparators given be the Heaviside step functions with the 'real' comparators, $\theta \rightarrow \mathcal{F}_{\Delta D}$.

Note that an arbitrary threshold domain can be represented by a combination (e.g., polynomial) of several threshold domains. For example, the cuboid given by equation (2) can be viewed as a product of six domains with plane boundaries, or as a product (intersection) of

10 two domains given by the rectangular cylinders

$$\mathcal{D}_{xy}(x, y) = \theta \left[1 - 4 \left(\frac{x - x_0}{a} \right)^2 \right] \theta \left[1 - 4 \left(\frac{y - y_0}{b} \right)^2 \right] \qquad (3)$$

and

$$\mathcal{D}_{yz}(y, z) = \theta \left[1 - 4 \left(\frac{y - y_0}{b} \right)^2 \right] \theta \left[1 - 4 \left(\frac{z - z_0}{c} \right)^2 \right], \qquad (4)$$

$\mathcal{D}(x, y, z) = \mathcal{D}_{xy}(x, y) \, \mathcal{D}_{yz}(y, z)$.

15 <u>Features of a signal</u> In terms of a threshold domain, a 'feature' of a signal is either a point inside of the domain, or a point on the boundary of the domain. For example, for the domain $\mathcal{D} = \theta[x(t) - D] \, \theta[\dot{x}(t)]$ (with two control levels, D and zero), a point *inside* (that is, $\mathcal{D} = 1$) corresponds to a positive-slope signal of the magnitude greater than D, and a point on the boundary of this domain is a stationary point of $x(t)$ above the threshold D.

20 One should notice that only a small fraction of the signal's trajectory might fall inside of the threshold domain, and thus the duration of the feature might be only a small fraction of the total duration of the signal, especially if a feature is defined as a point on the domain's boundary. Therefore, it is impractical to continuously digitize the signal in order to extract the desired short-duration features. To resolve this, ARTEMIS utilizes an analog technique

25 for extraction and quantification of the salient signal features.

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2 Analog counting

In its simplest form, analog counting consists of three steps: (1) time-differentiation, (2) rectification, and (3) integration. The result of step 2 (rectification) is the *instantaneous* count rate $\mathcal{R}(\mathbf{D}, t)$, and step 3 (integration) outputs the count rate $R(\mathbf{D}, t)$ in a moving window of

5   time $w(t)$, $R(\mathbf{D}, t) = w(t) * \mathcal{R}(\mathbf{D}, t)$.

Counting crossings of threshold domain boundaries   The number of crossings of the boundaries of a domain $\mathcal{D}$ by a point following the trajectory $\mathbf{x}(t)$ during the time interval $[0, T]$ can be written as

$$N = \int_0^T dt \left| \frac{d}{dt} \mathcal{D}[\mathbf{D}, \mathbf{x}(t)] \right| \tag{5}$$

10   for the total number of crossings, or

$$N_\pm = \int_0^T dt \left| \frac{d}{dt} \mathcal{D}[\mathbf{D}, \mathbf{x}(t)] \right|_\pm \tag{6}$$

for the number of entries $(+)$ or exits $(-)$. In equation (6), $|x|_\pm$ denotes positive/negative component of x,

$$|x|_\pm = \frac{1}{2} \left(|x| \pm x \right) . \tag{7}$$

15

Instantaneous count rates   Note that the integrands in equations (5) and (6) represent the *instantaneous* rates of crossings of the domain boundaries,

$$\mathcal{R}(t) = \sum_i \delta(t - t_i) , \tag{8}$$

where $\delta(t)$ is the Dirac delta function, and t_i are the instances of the crossings. It should be

20   easy to see that a number of other useful characteristics of the behavior of the signal inside the domain can be obtained based on the domain definition given by equation (1).

Consider, for example, a threshold domain $\mathcal{D}$ in a physical space given by a product

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(intersection) of two fields of view (e.g., solid angles) of two lidars[2] or cameras. When an object following the trajectory $\mathbf{x}(t)$ is in a field of view, the signal is '1'. Otherwise, it is '0'. Then the product of the signals from both lidars (cameras) is given by $\mathcal{D}[\mathbf{D}, \mathbf{x}(t)]$, and the counting of the crossings of the domain boundaries by the object can be performed by an

5   apparatus implementing equations (5) or (6). The following characteristics of the object's motion though the domain are also useful and easily obtained: The time spent inside the domain,

$$t = \int_0^T \mathrm{d}t\, \mathcal{D}[\mathbf{D}, \mathbf{x}(t)], \tag{9}$$

the distance traveled inside the domain,

10

$$s = \int_0^T \mathrm{d}t\, |\dot{\mathbf{x}}(\mathbf{t})|\, \mathcal{D}[\mathbf{D}, \mathbf{x}(t)], \tag{10}$$

and the average speed inside the domain,

$$\overline{v} = \frac{s}{t} = \frac{\int_0^T \mathrm{d}t\, |\dot{\mathbf{x}}(\mathbf{t})|\, \mathcal{D}[\mathbf{D}, \mathbf{x}(t)]}{\int_0^T \mathrm{d}t\, \mathcal{D}[\mathbf{x}(t)]}. \tag{11}$$

When using the 'real' comparators in a threshold filter and 'real' differentiators in an ana-

15   log counter, the main property of the 'real' instantaneous rate $\mathcal{R}(t)$ is

$$\lim_{\Delta D \to 0} \lim_{\delta t \to 0} \mathcal{R}(t) = \sum_i \delta(t - t_i), \tag{12}$$

where ΔD and δt are the width and the delay parameters of the comparators and differentiators, respectively. The property given by equation (12) determines the main uses of the instantaneous rate. For example, multiplication of the latter by a signal $x(t)$ amounts to sam-

20   pling this signal at the times of occurrence of the events t_i. Other temporal characteristics of the events can be constructed by time averaging various products of the signal with the instantaneous rate.

Count rate in a moving window of time   Count rate in a moving window of time $w_T(t)$ is obtained through the integration of the instantaneous rate by an integrator with an impulse

25   response $w_T(t)$, namely as

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[2]Here LIDAR is an acronym for "LIght Detector And Ranger".

20

$$R(\mathbf{D}, t) = w_T(t) * \mathcal{R}(\mathbf{D}, t).$$ (13)

Although there is effectively no difference between averaging window functions which rise from zero to a peak and then fall again, boxcar averaging is deeply engraved in modern engineering, partially due to the ease of interpretation and numerical computations. Thus one of the requirements for counting with a non-boxcar window is that the results of such measurements are comparable with boxcar counting. As an example, let us consider averaging of the instantaneous rates by a sequence of n RC-integrators. For simplicity, let us assume that these integrators have identical time constants $\tau = RC$, and thus their combined impulse response is

$$w_n(t) = \frac{t^{n-1}}{(n-1)! \, \tau^n} \, e^{-t/\tau} \, \theta(t).$$ (14)

Comparability with a boxcar function of the width T can be achieved by equating the first two moments of the respective weighting functions. Thus a sequence of n RC-integrators with identical time constants $\tau = \frac{1}{2} T/\sqrt{3n}$ will provide us with rate measurements corresponding to the time averaging with a rectangular moving window of width T.[3]

Figure 5 compares the moving window rates measured with the boxcar (thin solid line) and the 'triple-integrator' test function ($n = 3$ in equation (14), thick line) with $\tau = T/6$. The respective test functions are shown in the upper left corner of the figure. The gray band in the figure outlines the error interval in the rate measurements as the square root of the total number of counts in the time interval T per this interval.

One of the obvious shortcomings of boxcar averaging is that it does not allow meaningful differentiation of counting rates, while knowledge of time derivatives of the event occurrence rate is important for all physical models where such rate is a time-dependent parameter. Indeed, the time derivative of the rate measured with a boxcar function of width T is simply T^{-1} times the difference between the 'original' instantaneous pulse train and this pulse train

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[3]Of course one can design different criteria for equivalence of the boxcar weighting function and $w(t)$. In our example we were simply looking for the width parameter of $w(t)$ which would allow us to interpret the rate measurements with this function as 'a number of events per time interval T'.

21

delayed by T, and such representation of the rate derivative hardly provides physical insights. On the other hand, the time derivative of the 'cascaded integrators' weighting function w_n given by equation (14) is the bipolar pulse $\dot{w}_n = \tau^{-1}\left(w_{n-1} - w_n\right)$, and thus the time derivative of the rate evaluated with $w_n(t)$ is a measure of the 'disbalance' of the rates within the moving window (positive for a 'front-loaded' sample, and negative otherwise).

3   Multimodal pulse shaping

In order to focus upon characteristics of interest, feature definition may require knowledge of the (partial) derivatives of the signal. For example, in order to count the extrema in a signal $x(t)$, one needs to have access to the time derivative of the signal, $\dot{x}(t)$. A typical Multimodal Pulse Shaper in the present invention transforms at least one component of the incoming signal into at least two components such that one of these two components is a (partial) derivative of the other, and thus Multimodal Pulse Shaping can be used for embedding the incoming signal into a threshold space and enabling extraction of the features of interest by the Threshold Domain Filtering.

Note, however, that differentiation performed by any physical differentiator is not accurate. For example, a time derivative of $f(t)$ obtained by an RC differentiator is proportional to $\left[e^{-t/\tau}\theta(t)\right] * f(t)$, where $\tau = RC$, not to $\dot{f}(t)$. Thus Multimodal Pulse Shaping does not attempt to straightforwardly differentiate the incoming signal. Instead, it processes an incoming signal in parallel channels to obtain the necessary relations between the components of the output signal. For example, if $\mathbf{x}(\mathbf{a}, t)$ is a result of shaping the signal $\mathbf{y}(\mathbf{a}, t)$ by the first channel of a pulse shaper with the impulse response $f(\mathbf{a}, t)$,

$$\mathbf{x}(\mathbf{a}, t) = f(\mathbf{a}, t) * \mathbf{y}(\mathbf{a}, t), \tag{15}$$

then the derivatives of $\mathbf{x}$ can be obtained as

$$\dot{\mathbf{x}}(\mathbf{a}, t) = \dot{f} * \mathbf{y}(\mathbf{a}, t) \quad \text{and} \quad \frac{\partial}{\partial a_i}\mathbf{x}(\mathbf{a}, t) = \frac{\partial f}{\partial a_i} * \mathbf{y}(\mathbf{a}, t). \tag{16}$$

22

Thus Multimodal Pulse Shaping will be achieved if the impulse responses of various channels in the pulse shaper relate as the respective derivatives of the impulse response of the first channel.

Figure 6 shows an example of bimodal pulse shaping which can be used for both the
5 amplitude and timing measurements of a short-duration event. An event of magnitude E_i and arrival time t_i is passed through an RC pulse shaping network, producing a continuous signal $x(t)$. The event can be fully characterized, e.g., by the first extremum of $x(t)$, since the height of the extremum is proportional to E_i, and its position in time is delayed by a constant with respect to t_i. By replacing an RC integrator in the shaping network by an RC
10 differentiator with the same time constant, one can obtain an accurate time derivative of $x(t)$. Now the event can be associated with the inbound crossing of the boundary of the threshold domain $\mathcal{D} = \theta[x(t) - D]\ \theta[-\dot{x}(t)]$, where the threshold D is set at a positive value to eliminate the rest of the signal's stationary points.

3.1 Example: Analog Bimodal Coincidence (ABC) counting systems

15 Let us illustrate the usage of a threshold filter, in combination with multimodal pulse shaping and analog counting, in a signal processing module for a two-detector charged particle telescope. This module is an example of an Analog Bimodal Coincidence (ABC) counting system.

In our approach, we relate the short-duration particle events to certain *stationary* points
20 (e.g., local maxima) of a relatively slow analog signal. Those points can be accurately identified and characterized if the time derivative of the signal is available. Thus the essence of ABC counting systems is in the use of multiple signal characteristics – here a signal and its time derivative – and signals from multiple sensors in coincidence to achieve accuracy in both the amplitude and timing measurements while using low-speed, analog signal processing circuitry.
25 This allows us to improve both the engineering aspects of the instrumentation and the quality of the scientific data.

A simplified schematic of the module is shown in figure 7. A bimodal pulse shaping is used

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to obtain an accurate time derivative of the signal from a detector. Comparators are used to obtain two-level signals with the transitions at appropriate threshold crossings (e.g., zero crossings for the derivative signal). Simple asynchronous analog switches are used to obtain the products of the comparators' outputs suitable for appropriate conditional and coincidence counting. The comparators and the analog switches constitute the threshold domain filter with the thresholds $\{D_1\}$, $\{D_2\}$, and the grounds as the control levels. A-Counters are employed for counting the crossings of the threshold domains' boundaries. In its simplest form, an A-Counter is a differentiating circuit (such as a simple RC-differentiator) with a relatively small time constant (in order to keep the dead-time losses small), followed by a precision diode and an integrator with a large time constant (at least an order of magnitude larger than the inverted smallest rate to be measured). A TOF selector employs an additional pulse shaping amplifier, and a pair of comparators with the levels corresponding to the smallest and the largest time of flight.

Bimodal pulse shaping and instantaneous rate of signal's maxima   When the time derivative of a signal is available, we can relate the particle events to local maxima of the signal and accurately identify these events. Thus bimodal pulse shaping is the key to the high timing accuracy of the module. As shown in figure 7, a bimodal pulse shaping unit outputs *two* signals, where the second signal is proportional to an accurate time derivative of the first output. The rate $R(t)$, in the moving window of time $w_T(t)$, of a signal's maxima above the threshold D can be expressed as

$$R(t) = w_T(t) * \left| \frac{\mathrm{d}}{\mathrm{d}t} \left\{ \theta[x(t) - D] \, \theta[-\dot{x}(t)] \right\} \right|_+ , \qquad (17)$$

where $|y|_+$ denotes the positive part of y (see equation (7)), θ is the Heaviside unit step function, and the asterisk denotes convolution. Equation (17) represents an idealization of the measuring scheme consisting of the following steps: (i) the first output of the bimodal pulse shaping unit is passed through a comparator set at level D, and the second output – through a comparator set at zero level; (ii) the product of the outputs of the comparators is differentiated, (iii) rectified by a (precision) diode, and (iv) integrated on a time scale T (by an integrator with the impulse response $w_T(t)$). Note that steps (ii) through (iv) represent

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passing the product of the comparators through an A-Counter. Also note that the output of step (iii) is the *instantaneous* rate of the signal's maxima above threshold D.

Basic coincidence counting   For basic coincidence counting, the *coincident rate* $R_c(t)$ can be written as

5
$$R_c(t) = w_T(t) * \left| \frac{\mathrm{d}}{\mathrm{d}t} \{\theta[x_1(t) - D_1]\,\theta[-\dot{x}_1(t)]\,\theta[x_2(t) - D_2]\} \right|_+ , \qquad (18)$$

where the notations are as in equation (17). One can see that equation (18) differs from equation (17) only by an additional term in the product of the comparators' outputs.

Transition to realistic model of measurements   It can be easily seen that equations (17) and (18) do not correctly represent any practical measuring scheme implementable in hard-
10   ware. For example, both equations contain derivatives of discontinuous Heaviside functions, and thus instantaneous rates are expressed through singular Dirac δ-functions. To make a transition from an ideal measurement scheme to a more realistic model, we replace the Heaviside step functions by 'real' discriminators $(\theta(x) \rightarrow \alpha_{\delta t}(t) * \mathcal{F}_{\Delta D}(x)$, where $\alpha_{\delta t}(t)$ is a continuous kernel such that $\int_{-\infty}^{\infty}\mathrm{d}t\,\alpha_{\delta t}(t) = 1)$, and perform differentiation through a continuous kernel
15    $(\frac{\mathrm{d}}{\mathrm{d}t} \ldots \rightarrow \dot{\alpha}_{\delta t}(t) * \ldots)$, etc. We choose appropriate functional representations of $\mathcal{F}_{\Delta D}$, $\alpha_{\delta t}(t)$, etc., for various elements of a schematic, and also add appropriate noise sources such as thermal noise at all intermediate measuring steps. Figures 8 and 9 illustrate such realistic measurements of instantaneous rates of extrema and coincident maxima, respectively. Notice that, in both figures, an event is represented by a narrow peak of a prespecified area in the
20   instantaneous rates.

Time-of-flight (TOF) constrained measurements   The time-of-flight constrained coincident rate can be expressed, for times of flight larger than Δt, as

$$\mathcal{R}_c = w_T(t) * \left| \frac{\mathrm{d}}{\mathrm{d}t} \theta\left[h * |\mathcal{D}_{12} - \mathcal{D}_{21}|_+ - Z_{\Delta t} \right] \right|_+ , \qquad (19)$$

where h is some (unipolar or bipolar) impulse response function, $Z_{\Delta t}$ is a threshold level
25   corresponding to the TOF equal to Δt, and $D_{ij} = |\theta\,[x_i(t) - D_i]\,\theta\,[-\dot{x}_i(t)]\,\theta\,[x_j(t) - D_j]|_+$. Thus a TOF selector (see figure 7) will consist of a pulse shaping amplifier with an impulse response h, and a differential comparator. Figure 10(a) illustrates coincident counting

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according to equation (19), and figure 10(b) provides an example of using a realistic model of the TOF measurements, with functional representations of the elements of the schematic corresponding to commercially-available, off-the-shelf (COTS) components. As can be seen in the figure, the performance of the system is not significantly degraded by the transition

5   from an idealized to a more realistic model.

4   Analog Rank Filtering

In ARTEMIS, Analog Rank Filtering can be used for establishing and maintaining the analog control levels of the Threshold Domain Filtering. It ensures the adaptivity of the Threshold

10   Domain Filtering to changes in the measurement conditions (e.g., due to nonstationarity of the signal or instrument drift), and thus the optimal separation of the features of interest from the rest of the signal. For example, the threshold level D in the domain $\mathcal{D} = \theta[x(t) - D]\ \theta[\dot{x}(t)]$ can be established by means of Analog Rank Filtering to separate the stationary points of interest from those caused by noise. Note that the Analog Rank Filtering outputs the control levels

15   indicative of the salient properties of the input signal(s), and thus can be used as a stand-alone embodiment of ARTEMIS for adaptive real-time signal conditioning, processing, analysis, quantification, comparison, and control, and for detection, quantification, and prediction of changes in signals.

Creating and maintaining baseline and analog control levels by analog rank filters   *Analog*

20   *rank filters* can be used to establish various control levels (reference thresholds) for the threshold filter. When used in ARTEMIS, rank-based filters allows us to reconcile, based on the rank filters' insensitivity to outliers, the conflicting requirements of the robustness and adaptability of the control levels of the Threshold Domain Filtering. In addition, the control levels created by Analog Rank Filters are themselves indicative of the salient properties of the input

25   signal(s).

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4.1   Adaptive Analog Rank Filters (AARFs)

Rank filter in RC window When the time averaging filter in equation (D-3) is an RC integra-tor $(RC = \tau)$, the differential equation for the output $D_q(t)$ of a rank filter takes an especially simple form and can be written as

$$\frac{dD_q}{dt} = \frac{A(2q-1) - \tilde{\mathcal{F}}_{\Delta D}\left[D_q(t) - x(t)\right]}{2A\,\tau\,h_\tau(s) * f_{\Delta D}\left[D_q(t) - x(s)\right]\Big|_{s=t}}, \tag{20}$$

where $h_\tau(t) = \theta(t)\exp\left(-\frac{t}{\tau} - \ln\tau\right).$[4] The solution of this equation is ensured to rapidly converge to $D_q(t)$ of the chosen quantile order q regardless of the initial condition (Nikitin and Davidchack, 2003b). Note also that the continuity of the comparator is essential for the right-hand side of equation (20) to be well behaved.

The main obstacle to a straightforward analog implementation of the filter given by equa-tion (20) is that the convolution integral in the denominator of the right-hand side needs to be re-evaluated (updated) for each new value of D_q. If we wish to implement an analog rank filter in a simple feedback circuit, then we should replace the right-hand side of equation (20) by an approximation which can be easily evaluated by such a circuit. Of course, one can employ a great variety of such approximations (Bleistein and Handelsman, 1986, for example), whose suitability will depend on a particular goal. A very simple approximation becomes available in the limit of sufficiently small τ, since then we can replace $h_\tau(s) * f_{\Delta D}\left[D_q(t) - x(s)\right]\Big|_{s=t}$ by $h_\tau(t) * f_{\Delta D}\left[D_q(t) - x(t)\right]$ in equation (20). As was shown by Nikitin and Davidchack (2003b), this simple approximation can still be used for an arbitrary time window $w(t)$, if we repre-sent $w(t)$ as a weighted sum of many RC integrators with small τ. However, this approximation fails when the threshold resolution is small (e.g., when $\Delta D < |h_\tau(t) * \dot{x}(t)|\,\tau$), and thus cannot be used in real-time processing of non-stationary signals.

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[4]In more explicit notation, the convolution integral in the denominator of equation (20) can be written as

$$h_\tau(s) * f_{\Delta D}\left[D_q(t) - x(s)\right]\Big|_{s=t} = \frac{1}{\tau}\int_{-\infty}^{t}ds\,\exp\left(\frac{s-t}{\tau}\right)f_{\Delta D}\left[D_q(t) - x(s)\right].$$

Adaptive approximation of a feedback rank filter in an arbitrary time window A rank filter
in a boxcar moving time window $B_T(t) = [\theta(t) - \theta(t - T)]/T$ is of a particular interest, since
it is the most commonly used window in digital rank filters. The output D_q of an analog
rank filter in this window is implicitly defined as $B_T(t) * \mathcal{F}_{\Delta D}[D_q - x(t)] = q$. To construct
5   an approximation for this filter suitable for implementation in an analog feedback circuit, we
first approximate the boxcar window $B_T(t)$ by the following moving window $w_N(t)$:[5]

$$w_N(t) = \frac{1}{N} \sum_{k=0}^{N-1} h_\tau(t - 2k\tau), \tag{21}$$

where $\tau = T/(2N)$. The first moments of the weighting functions $w_N(t)$ and $B_T(t)$ are
identical, and the ratio of their respective second moments is $\sqrt{1 + 2/N^2} \approx 1 + 1/N^2$. The other
10 moments of the time window $w_N(t)$ also converge rapidly, as N increases, to the respective
moments of $B_T(t)$, which justifies the approximation of equation (21).

Now, the output of a rank filter in such a window can be approximated as discussed earlier,
namely as (Nikitin and Davidchack, 2003b)[6]

$$\frac{dD_q}{dt} \approx \frac{A\,N(2q - 1) - \sum_{k=0}^{N-1} \tilde{\mathcal{F}}_{\Delta D}[D_q(t) - x(t - 2k\tau)]}{2A\,\tau\,h_\tau(t) * \sum_{k=0}^{N-1} f_{\Delta D}[D_q(t) - x(t - 2k\tau)]}, \tag{22}$$

15 where $\tau = T/(2N)$. Note that the accuracy of this approximation is contingent on the re-
quirement that $\Delta D > |h_\tau(t) * \dot{x}(t)|\,\tau$. This means that, if we wish to have a simple analog
circuit and keep N relatively small, we must choose ΔD sufficiently large for the approxima-
tion to remain accurate. On the other hand, we would like to maintain high resolution of the
acquisition system, that is, to keep ΔD small.

20 In order to reconcile these conflicting requirements, we propose to use an *adaptive* approxi-
mation, which reduces the resolution only when necessary. This can be achieved, for example,
by using equation (D-2) and rewriting the threshold derivative of $h_\tau(t) * \tilde{\mathcal{F}}_{\Delta D}[D_q - x(t)]$ as

---

[5]Since a moving time window is always a part of a convolution integral, the approximation is understood
in the sense that $B_T(t) * g(t) \approx w_N(t) * g(t)$, where $g(t)$ is a smooth function.

[6]An explicit expression for the convolution integral $h_\tau(t) * f_{\Delta D}[D_q(t) - x(t - 2k\tau)]$ is

$$h_\tau(t) * f_{\Delta D}[D_q(t) - x(t - 2k\tau)] = \frac{1}{\tau} \int_{-\infty}^{t} ds \, \exp\left(\frac{s - t}{\tau}\right) f_{\Delta D}[D_q(s) - x(s - 2k\tau)].$$

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Inventor: Nikitin
US Express: EK948840967US

$$h_\tau(t) * f_{\Delta D}\left[D_q - x(t)\right] \approx \frac{h_\tau(t) * \left\{\widetilde{\mathcal{F}}_{\Delta D}\left[D_{q+} - x(t)\right] - \widetilde{\mathcal{F}}_{\Delta D}\left[D_{q-} - x(t)\right]\right\}}{4A\left(D_{q+} - D_{q-}\right)}, \tag{23}$$

where $D_{q\pm}$ is the output of a rank filter of the quantile order $q \pm \delta q$, $\delta q \ll q$. In essence, the approximation of equation (23) amounts to decreasing the resolution of the acquisition system only when the amplitude distribution of the signal broadens, while otherwise retaining high resolution.

Combining equations (21–23), we arrive at the following representation of an adaptive approximation to a feedback rank filter in a boxcar time window of width T:

$$
\begin{aligned}
\dot{D}_q(t) &= \frac{1}{2}\left[D_{q+}(t) + D_{q-}(t)\right] \\
\dot{D}_{q+}(t) &= \frac{A\,N(2q - 1 + 2\delta q) - \sum_{k=0}^{N-1}\widetilde{\mathcal{F}}_{\Delta D}\left[D_{q+}(t) - x(t - 2k\tau)\right]}{h_\tau(t) * \delta\widetilde{\mathcal{F}}_{\Delta D}(t)}\frac{\delta D_q(t)}{\tau}, \\
\dot{D}_{q-}(t) &= \frac{A\,N(2q - 1 - 2\delta q) - \sum_{k=0}^{N-1}\widetilde{\mathcal{F}}_{\Delta D}\left[D_{q-}(t) - x(t - 2k\tau)\right]}{h_\tau(t) * \delta\widetilde{\mathcal{F}}_{\Delta D}(t)}\frac{\delta D_q(t)}{\tau}
\end{aligned}
\tag{24}
$$

where $\delta D_q(t) = D_{q+}(t) - D_{q-}(t)$ and

$$\delta\widetilde{\mathcal{F}}_{\Delta D}(t) = \sum_{k=0}^{N-1}\left\{\widetilde{\mathcal{F}}_{\Delta D}\left[D_{q+}(t) - x(t - 2k\tau)\right] - \widetilde{\mathcal{F}}_{\Delta D}\left[D_{q-}(t) - x(t - 2k\tau)\right]\right\}. \tag{25}$$

This approximation preserves its validity for high resolution comparators (small ΔD), and its output converges, as N increases, to the output of the 'exact' rank filter in the boxcar time window $B_T(t)$. Unlike the currently known approaches (see, for example, Urahama and Nagao 1995; Opris 1996), the analog rank filters enabled through equation (24) are not constrained by linear convergence and allow real time implementation on an arbitrary timescale, thus enabling high speed real time rank filtering by analog means. The accuracy of this approximation is best described in terms of the error in the quantile q. That is, the output $D_q(t)$ can be viewed as bounded by the outputs of the 'exact' rank filter for different quantiles $q \pm \Delta q$. When ΔD and δq in equation (24) are small, the error range Δq is of order $1/N$.

Note that, even though equation (24) represents a *feedback* implementation of a rank filter, it is *stable* with respect to the quantile values q. In other words, the solution of this equation will rapidly converge to the 'true' value of $D_q(t)$ regardless of the initial condition, and the time of convergence within the resolution of the filter ΔD for any initial condition will be just a

small fraction of τ. This convergence property is what makes the implementation represented by equation (24) suitable for a real time operation on an arbitrary timescale.

Implementation of AARFs in analog feedback circuits   Figure 11 illustrates implementation of an adaptive real time rank filter given by equation (24) in an analog feedback circuit. One

5   skilled in the art will recognize that this circuit is a simplified embodiment of a more general AARF depicted in figure 12.

Generalized description of AARFs   As shown in figure 12, an input variable $x(t)$ and a plurality of feedbacks of Offset Rank Filtered Variables $\{D_{q_i}(t)\}$ are passed through a plurality of (delayed) comparators forming a plurality of outputs of the comparators $\{\widetilde{\mathcal{F}}_i^{\mathrm{del}}(t)\}$ =

10    $\{\widetilde{\mathcal{F}}_{\Delta D}^{\mathrm{del}}[D_{q_i}(t), x(t)]\}$. (Please note that in this and further figures a double line in a diagram indicates a plurality of signals.)   Said plurality of the outputs of the comparators $\{\widetilde{\mathcal{F}}_i^{\mathrm{del}}\}$ is used to form (i) a plurality $\{A(2q_i - 1) - \widetilde{\mathcal{F}}_i^{\mathrm{del}}\}$ of differences between said outputs of the comparators and the respective Offset Quantile Parameters of said Offset Rank Filtered Variables, and (ii) a weighted difference $\delta\widetilde{\mathcal{F}}_{\Delta D}(t) = \sum_i \alpha_i \widetilde{\mathcal{F}}_i^{\mathrm{del}}$, where $\sum_i \alpha_i = 0$, of said outputs

15   of the comparators.   Said weighted difference $\delta\widetilde{\mathcal{F}}_{\Delta D}(t)$ of the outputs of the comparators is passed through a time averaging amplifier, forming a Density Function $h_\tau(t) * \delta\widetilde{\mathcal{F}}_{\Delta D}(t)$. The plurality of the feedbacks of the Offset Rank Filtered Variables $\{D_{q_i}(t)\}$ is used to form a weighted difference $\delta D_q(t) = \sum_i \beta_i D_{q_i}(t)$, where $\sum_i \beta_i = 0$, of said feedbacks.   Each difference $A(2q_i - 1) - \widetilde{\mathcal{F}}_i^{\mathrm{del}}$ between the outputs of the comparators and the respective Offset Quan-

20   tile Parameters of the Offset Rank Filtered Variables is multiplied by a ratio of the weighted difference $\delta D_q(t)$ of the feedbacks of the Offset Rank Filtered Variables and the Density Function $h_\tau(t) * \delta\widetilde{\mathcal{F}}_{\Delta D}(t)$, forming a plurality of time derivatives of Offset Rank Filtered Variables $\{\dot{D}_{q_i}(t)\}$.   Said plurality of the time derivatives $\{\dot{D}_{q_i}(t)\}$ is integrated to produce the plurality of the Offset Rank Filtered Variables $\{D_{q_i}(t)\}$.   The plurality of the Offset Rank Filtered

25   Variables $\{D_{q_i}(t)\}$ is then used to form an output Rank Filtered Variable $D_q(t)$ as a weighted average $\sum_i w_i D_{q_i}(t)$, $\sum_i w_i = 1$, of said Offset Rank Filtered Variables.

As an example, figure 13 provides a simplified diagram of a 3-comparator implementation of AARF. In this example, the offset quantile orders are $q_2 = q$, $q_1 = q - dq$, and $q_3 = q + dq$, and the weights for weighted average and differences are: $w_2 = 1$, $w_1 = w_3 = 0$, $\alpha_2 = \beta_2 = 0$,

and $\alpha_1 = \beta_1 = -\alpha_3 = -\beta_3 = -1$.

Note that both the input and output of an AARF are continuous signals. The width of the moving window and the quantile order are continuous parameters as well, and such continuity can be utilized in various analog control systems. The adaptivity of the approximation allows

5   us to maintain a high resolution of the comparators regardless of the properties of the input signal, which enables the usage of this filter for nonstationary signals.

Also, let us point out that the equations describing this filter are also suitable for numerical computations, especially when the number of data points within the moving window is large. A simple forward Euler method is fully adequate for integrating these equations, and the

10 numerical convolution with an RC impulse response function requires remembering only one previous value. Thus numerical algorithms based on these equations have the advantages of both high speed and low memory requirements.

Delayed comparators   In our description of AARFs we have assumed that the comparators are the *delayed comparators* with the outputs represented by the moving averages

15
$$\widetilde{\mathcal{F}}_{\Delta D}^{\mathrm{del}}\left[D_q(t), x(t)\right] = \sum_{k=0}^{N-1} w_k\, \widetilde{\mathcal{F}}_{\Delta D}\left[D_q(t) - x(t - \Delta t_k)\right], \tag{26}$$

where w_k are positive weights such that $\sum_k w_k = 1$, and it can be assumed, without loss of generality, that $\Delta t_0 = 0$. Obviously, when $N = 1$, a delayed comparator is just a simple two-level comparator. Figure 14 illustrates a principle schematic of a delayed comparator.

Averaging comparators   In the description of Adaptive Analog Rank Selectors further in this

20 disclosure we will use another type of a comparator, which we refer to as an *averaging com-parator*. Unlike a delayed comparator which takes a threshold level and a scalar signal as inputs, the inputs of an averaging comparator are a threshold level D and a *plurality* of input signals $\{x_i(t)\}$, $i = 1, \ldots, N$. The output of an averaging comparator is then given by the expression

25
$$\widetilde{\mathcal{F}}_{\Delta D}^{\mathrm{ave}}\left[D_q(t), \{x_i(t)\}\right] = \sum_{i=1}^{N} w_i\, \widetilde{\mathcal{F}}_{\Delta D}\left[D_q(t) - x_i(t)\right], \tag{27}$$

where w_i are positive weights such that $\sum_i w_i = 1$. Figure 15 illustrates a principle schematic of an averaging comparator.

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Figure 16 compares the performance of the analog rank filter given by equation (24) to that of the 'exact' quantile filter in a boxcar moving window of width T. In this example, the quantile interval δq is chosen as $\delta q = 10^{-2}$ (1%). The continuous input signal $x(t)$ (shown by the solid dark gray line) is emulated as a high resolution time series (2×10^3 points per

5   interval T). The 'exact' outputs of a boxcar window rank filter are shown by the dashed lines, and their deviations within the $\pm \Delta q$ intervals are shown by the gray bands. The respective outputs of the approximation given by equation (24) are shown by the solid black lines. The width parameter ΔD of the comparators, the width T of the boxcar time window, the quantile order q, and the number N of exponential kernels in the approximation are indicated in the

10   figure.

The (instantaneous) accuracy of the approximation given by equation (24) decreases when the input signal $x(t)$ undergoes a large (in terms of the resolution parameter ΔD) monotonic change over a time interval of order τ. The main effect of such a 'sudden jump' in the input signal is to delay the output $D_q(t)$ relative to the output of the respective 'exact' filter. This

15   delay is shown as Δt in the lower left portion of the upper panel, where the input signal is a square pulse. This timing error Δt is inversely proportional to the number N of the kernels in the approximation. The accuracy of the approximation can also be described in terms of the *amplitude* error. As can be seen in figure 16, the residual oscillations of the outputs of the analog filter occur within the $q \pm 1/(2N)$ interval around the respective outputs of the 'exact'

20   filter (that is, within the width of the gray bands in the figure).

Establishing internal reference signal (baseline and analog control levels) As stated earlier, a

primary use of Analog Rank Filtering in ARTEMIS is establishing and maintaining the analog

control levels of the Threshold Domain Filtering, which ensures the adaptivity of the Threshold

Domain Filtering to changes in the measurement conditions, and thus the optimal separation

5   of the features of interest from the rest of the signal.   Such robust control levels can be

established, for example, by filtering the components of the signal with a Linear Combination

of Analog Order Statistics Filters operable on a given timescale.

Example: 'Trimean' reference. Figure 17 provides an example of using an internal reference

(baseline) for separating signal from noise, and illustrates a technique for establishing a refer-

10   ence baseline as a linear combination of quartile outputs (i.e., $q = 1/4$, $q = 1/2$, and $q = 3/4$)

of AARFs. In this example, the features of interest are tall pulses protruding from a noisy

background. For example, one would want to count the number of such pulses, while ignoring

the smaller pulses due to noise. This can be accomplished by choosing a reference baseline

such that most of the pulses of interest peak above this baseline, while the accidental crossings

15   of the baseline by noise are rare. A good choice for a baseline thus would be a moving average

of the noise plus several standard deviations of the noise in the same moving window of time

(a 'variance' baseline, gray lines in the figure). However, the presence of the high-amplitude

pulses of the 'useful' signal will significantly disturb such a baseline. Instead, one can create a

baseline by using a linear combination of the outputs of AARFs for different quantile orders

20   (e.g., for the quartiles $q = 1/4$, $q = 1/2$, and $q = 3/4$ – 'quartile' baseline, dashed lines in

the figure). As shown in the upper panel of the figure, in the absence of the signal of interest

the baselines created by both techniques are essentially equivalent. However, as shown in the

lower panel of the figure, in the presence of tall pulses the 'variance' baseline is significantly

disturbed and fails to separate the noise from the signal, while the 'quartile' baseline remains

25   virtually unaffected by the addition of these pulses. In both panels, the distance between the

time ticks is equal to the width of the moving time window.

Figure 18 illustrates a principle diagram of a circuit for establishing a baseline as a linear

combination of the quartile outputs ($q = 1/4$, $q = 1/2$, and $q = 3/4$) of AARFs. One skilled

in the art will recognize that a variety of other linear combinations of outputs of AARFs of

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different quantile orders can be used for establishing and maintaining the analog control levels of the Threshold Domain Filtering.

Single Point Analog Rank Tracker (SPART) The approximation of equation (24) preserves its validity for high resolution comparators (small ΔD), and its output converges, as N increases,

5   to the output of the 'exact' rank filter in the boxcar time window $B_T(t)$. However, even a single-point approximation ($N = 1$ in equation (24), i.e., simple rather than delayed comparators in AARF) can be fully adequate for creating and maintaining the baseline and analog control levels in analog counting systems, since such a simplified implementation preserves the essential properties of the 'exact' rank filter needed for this purpose. We shall call this version of an

10   AARF the 'Single Point Analog Rank Tracker', or SPART.

Adaptive Analog Rank Selectors (AARSs) While an AARF operates on a single scalar input signal $x(t)$ and outputs a q th quantile $D_q(t)$ of the input signal in a moving window of time, an AARS operates on a *plurality* of input signals $\{x_i(t)\}$, $i = 1, \ldots, N$, and outputs ('selects') an *instantaneous* q th quantile $D_q(t)$ (in general, a *weighted* quantile) of the plurality of the input signals. Such transition from an AARF to an AARS can be achieved by replacing the delayed comparators in an AARF by averaging comparators. For example, a 2-comparator AARS can be represented by the following equation:

$$
\begin{aligned}
\dot{D}_q(t) &= \frac{1}{2}\left[D_{q+}(t) + D_{q-}(t)\right] \\
\dot{D}_{q+}(t) &= \frac{A\,N(2q - 1 + 2\delta q) - \sum_{i=1}^{N} \widetilde{\mathcal{F}}_{\Delta D}\left[D_{q+}(t) - x_i()\right]}{h_\tau(t) * \delta\widetilde{\mathcal{F}}_{\Delta D}(t)} \frac{\delta D_q(t)}{\tau} \\
\dot{D}_{q-}(t) &= \frac{A\,N(2q - 1 - 2\delta q) - \sum_{i=1}^{N} \widetilde{\mathcal{F}}_{\Delta D}\left[D_{q-}(t) - x_i(t)\right]}{h_\tau(t) * \delta\widetilde{\mathcal{F}}_{\Delta D}(t)} \frac{\delta D_q(t)}{\tau}
\end{aligned}
\qquad (28)
$$

where $\delta D_q(t) = D_{q+}(t) - D_{q-}(t)$ and

$$
\delta\widetilde{\mathcal{F}}_{\Delta D}(t) = \sum_{i=1}^{N}\left\{\widetilde{\mathcal{F}}_{\Delta D}\left[D_{q+}(t) - x_i(t)\right] - \widetilde{\mathcal{F}}_{\Delta D}\left[D_{q-}(t) - x_i(t)\right]\right\}. \qquad (29)
$$

Note that the time of convergence (or *time of rank selection*) is proportional to the time constant $\tau = RC$ of the RC integrator, and thus can be made sufficiently small for a true real time operation of an AARS. Figure 19 illustrates a principle schematic of an Adaptive Analog Rank Selector given by equation (28) in an analog feedback circuit. One skilled in the art will recognize that this circuit is a simplified embodiment of a more general AARS depicted in figure 20.

Generalized description of AARSs As shown in figure 20, a plurality input variables $\{x_j(t)\}$, $j = 1, \ldots, N$, and a plurality of feedbacks of Offset Rank Selected Variables $\{D_{q_i}(t)\}$ are passed through a plurality of averaging comparators forming a plurality of outputs of the comparators $\{\widetilde{\mathcal{F}}_i^{\text{ave}}(t)\} = \{\widetilde{\mathcal{F}}_{\Delta D}^{\text{ave}}\left[D_{q_i}(t), x_j(t)\right]\} = \{\sum_{j=1}^{N} v_j\, \widetilde{\mathcal{F}}_{\Delta D}\left[D_{q_i}(t) - x_j(t)\right]\}$. Said plurality of the outputs of the comparators $\{\widetilde{\mathcal{F}}_i^{\text{ave}}\}$ is used to form (i) a plurality $\{A(2q_i - 1) - \widetilde{\mathcal{F}}_i^{\text{ave}}\}$ of differences between said outputs of the comparators and the respective Offset Quantile Parameters of said Offset Rank Selected Variables, and (ii) a weighted difference $\delta\widetilde{\mathcal{F}}_{\Delta D}(t) = \sum_i \alpha_i \widetilde{\mathcal{F}}_i^{\text{ave}}$, where $\sum_i \alpha_i = 0$, of said outputs of the comparators. Said weighted difference

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$\delta\widetilde{\mathcal{F}}_{\Delta D}(t)$ of the outputs of the comparators is passed through a time averaging amplifier, forming a Density Function $h_\tau(t) * \delta\widetilde{\mathcal{F}}_{\Delta D}(t)$. The plurality of the feedbacks of the Offset Rank Selected Variables $\{D_{q_i}(t)\}$ is used to form a weighted difference $\delta D_q(t) = \sum_i \beta_i D_{q_i}(t)$, where $\sum_i \beta_i = 0$, of said feedbacks. Each difference $A(2q_i - 1) - \widetilde{\mathcal{F}}_i^{\text{ave}}$ between the outputs of

5   the comparators and the respective Offset Quantile Parameters of the Offset Rank Selected Variables is multiplied by a ratio of the weighted difference $\delta D_q(t)$ of the feedbacks of the Offset Rank Selected Variables and the Density Function $h_\tau(t) * \delta\widetilde{\mathcal{F}}_{\Delta D}(t)$, forming a plurality of time derivatives of Offset Rank Selected Variables $\{\dot{D}_{q_i}(t)\}$. Said plurality of the time derivatives $\{\dot{D}_{q_i}(t)\}$ is integrated to produce the plurality of the Offset Rank Selected Variables $\{D_{q_i}(t)\}$.

10   The plurality of the Offset Rank Selected Variables $\{D_{q_i}(t)\}$ is then used to form an output Rank Selected Variable $D_q(t)$ as a weighted average $\sum_i w_i D_{q_i}(t)$, $\sum_i w_i = 1$, of said Offset Rank Selected Variables.

Adaptive Analog Rank Selectors are well suited for analysis and conditioning of spatially-extended objects such as multidimensional images. For example, a plurality of input signals

15   can be the plurality of the signals from a vicinity around the spatial point of interest, and the weights $\{v_j\}$ can correspond to the weights of a spatial averaging kernel. This enables us to design highly efficient real-time analog rank filters for removing dynamic as well as static impulse noise from an image, as illustrated in figure 21 for a two-dimensional monochrome image. In this example, a median filter ($q = 1/2$) according to equation (28) is

20   used.[7] Panel (a) shows the original (uncorrupted) image. Panel (b) shows the snapshots, at different times, of the noisy image and the respective outputs of the filter. In this example, approximately 4/5 of the pixels of the original image are affected by a bipolar non-Gaussian random noise at any given time. Panel (c) provides an example of removing the static noise (1/3 of the pixels of the original image are affected). This example also illustrates the fact

25   that the characteristic time of convergence of the filter based on equation (28) is only a small fraction of the time constant $\tau = RC$ of the RC integrator, which makes this circuit suitable for a truly real-time operation. This fast convergence is a consequence of the fact that the

---

[7]In general, the quantile order of the filter should be chosen as $q = \Phi_n(0)$, where Φ_n is the amplitude distribution of the noise (either measured or known *a priori*). In the example of this section, $\Phi_n(0) = 1/2$.

speed of convergence is inversely proportional to the density function $h_\tau(t) * \delta \widetilde{\mathcal{F}}_{\Delta D}(t)$.

4.2 Explicit Analog Rank Locators (EARLs)

Explicit expression for an analog quantile filter Note that a differential equation is not the

5   only possible embodiment of an analog quantile filter. Other means of locating the level lines

of the threshold distribution function can be developed based on the geometric interpretation

discussed in §D-2. For example, one can start by using the sifting property of the Dirac δ-

function to write $D_q(t)$ as

$$D_q(t) = \int_{-\infty}^{\infty} \mathrm{d}D \, D \, \delta[D - D_q(t)] \tag{30}$$

10   for all t. Then, recalling that $D_q(t)$ is a root of the function $\Phi(D, t) - q$ and that, by con-

struction, there is only one such root for any given time t, we can replace the δ-function of

thresholds with that of the distribution function values as follows:

$$D_q(t) = \int_{-\infty}^{\infty} \mathrm{d}D \, D \, \phi(D, t) \, \delta \left[\Phi(D, t) - q \right] . \tag{31}$$

Here we have used the following property of the Dirac δ-function (see Davydov, 1988, p. 610,

15   eq. (A 15), for example):

$$\delta[a - f(x)] = \sum_i \frac{\delta(x - x_i)}{|f'(x_i)|}, \tag{32}$$

where $|f'(x_i)|$ is the absolute value of the derivative of $f(x)$ at x_i, and the sum goes over all

x_i such that $f(x_i) = a$. We have also used the fact that $\phi(D, t) \geq 0$.

The final step in deriving a practically useful realization of the quantile filter is to replace

20   the δ-function of the ideal measurement process with a finite-width pulse function $g_{\Delta q}$ of the

real measurement process, namely

$$D_q(t) = \int_{-\infty}^{\infty} \mathrm{d}D \, D \, \phi(D, t) \, g_{\Delta q} \left[\Phi(D, t) - q \right] , \tag{33}$$

where Δq is the characteristic width of the pulse. That is, we replace the δ-function with a

continuous function of finite width and height. This replacement is justified by the observation

made earlier: it is impossible to construct a physical device with an impulse response expressed by the δ-function, and thus an adequate description of any real measurement must use the actual response function of the acquisition system instead of the δ-function approximation. We shall call an analog rank filter given by equation (33) the *Explicit Analog Rank Locator*

5   *(EARL)*.

Analog L filters and α-trimmed mean filters   It is worth pointing out the generalization of analog quantile filters which follows from equation (31). In the context of digital filters, this generalization corresponds to the L filters described by Bovik et al. (1983).

Indeed, we can write a linear combination of the outputs of various quantile filters as

$$
\begin{aligned}
D_L(t) = \int_0^1 \mathrm{d}q\, W_L(q)\, D_q(t) &= \int_0^1 \mathrm{d}q\, W_L(q) \int_{-\infty}^{\infty} \mathrm{d}D\, D\, \phi(D,t)\, \delta\left[\Phi(D,t) - q\right] \\
&= \int_{-\infty}^{\infty} \mathrm{d}D\, D\, \phi(D,t)\, W_L\left[\Phi(D,t)\right] ,
\end{aligned} \tag{34}
$$

where W_L is some (normalized) weighting function. Note that the difference between equations (34) and (33) is in replacing the narrow pulse function $g_{\Delta q}$ in (33) by an *arbitrary* weighting function W_L.

A particular choice of W_L in (34) as the rectangular (boxcar) probe of width $1 - 2\alpha$,

15 centered at $1/2$, will correspond to the digital α-trimmed mean filters described by Bendat (1998):

$$
\overline{D}_\alpha(t) = \int_{-\infty}^{\infty} \mathrm{d}D\, D\, \phi(D,t)\, b_\alpha\left[\Phi(D,t)\right] , \qquad 0 \le \alpha < 1/2 , \tag{35}
$$

where $b_\alpha(x) = \frac{1}{1-2\alpha}\left[\theta(x - \alpha) - \theta(x - 1 + \alpha)\right]$. When $\alpha = 0$, equation (35) describes the running mean filter, $\overline{D}_{\alpha=0}(t) = \overline{x}(t)$, and in the limit $\alpha \to 1/2$ it describes the median filter,

20 $\lim_{\alpha \to 1/2} \overline{D}_\alpha(t) = D_m(t)$.

Dealing with improper integration: Adaptive EARL   The main practical shortcoming of the filter given by equation (33) is the improper integral with respect to threshold. This difficulty, however, can be overcome by a variety of ways.

For example, we can use the fact that rank is not affected by a monotonic trans-

25 formation. That is, if D_q is the qth quantile of the distribution $w_r(t) * \theta\left[D - x(t)\right]$

(that is, $w_\tau(t) * \theta[D_q - x(t)] = q$), then $\mathcal{F}(D_q)$ is the qth quantile of the distribution $w_\tau(t) * \theta\{\mathcal{F}(D) - \mathcal{F}[x(t)]\}$:

$$w_\tau(t) * \theta\{\mathcal{F}(D_q) - \mathcal{F}[x(t)]\} = q\,, \tag{36}$$

where $\mathcal{F}(\xi)$ is a monotonically increasing function of ξ.

Now let us choose $\overline{\xi} = \mathcal{F}(\xi)$ as the response of a real comparator, $\mathcal{F}(\xi) = \mathcal{F}_{\mu_2}(\xi - \mu_1)$, where μ_1 is indicative of the mean value of $x(t)$ in a moving window w_T of the width T much greater than τ, and the width parameter μ_2 is indicative of the signal's deviation around μ_1 (on a similar time interval). For example,

$$\begin{cases} \overline{\xi} = \mathcal{F}_{\mu_2}(\xi - \mu_1) \\ \mu_1(t) = w_T(t) * x(t) \\ \mu_2(t) = \sqrt{2\left[w_T(t) * x^2(t) - \mu_1^2(t)\right]} \end{cases}. \tag{37}$$

Then an equation for the adaptive explicit analog rank locator can be rewritten as

$$D_q(t) = \mu_1(t) + \mathcal{F}_{\mu_2}^{-1}\left\{\int_0^1 \mathrm{d}\chi\, \chi\, \tilde{\phi}(\chi, t)\, g_{\Delta_q}\left[\tilde{\Phi}(\chi, t) - q\right]\right\}, \tag{38}$$

where

$$\tilde{\phi}(\chi, t) = \frac{w_\tau(t) * \{K(t)\, f_{\Delta_D}[\chi - \overline{x}(t)]\}}{w_\tau(t) * K(t)}, \tag{39}$$

and

$$\tilde{\Phi}(\chi, t) = \frac{w_\tau(t) * \{K(t)\, \mathcal{F}_{\Delta_D}[\chi - \overline{x}(t)]\}}{w_\tau(t) * K(t)}. \tag{40}$$

Note that the improper integral of equation (33) has become an integral over the finite interval $[0, 1]$, where the variable of integration is a dimensionless variable χ.

Figure 22 illustrates the performance of adaptive EARLs operating as amplitude (panel b)) and counting (panel (c)) rank filters in comparison with the 'exact' outputs of the respective analog rank filters given by equation (D-6).

Discrete-Threshold Approximation to Adaptive EARL Given a monotonic array of threshold values between zero and unity, the integral in equation (38) can be evaluated in finite differences leading to a discrete-threshold approximation to adaptive EARL as follows:

39

$$D_q(t) = \mu_1(t) + \mathcal{F}_{\mu_2}^{-1}(\overline{D}_q),\tag{41}$$

where $\overline{D}_q$ is the root of $\tilde{\Phi}(D,t) = q$. For example, $\overline{D}_q = \frac{1}{2}(D_{j_1} + D_{j_2})$, where $0 \le D_i \le 1$ is a monotonic array of threshold values, $D_i < D_{i+1}$, and j_1 and j_2 are such that $\tilde{\Phi}(D_{j_1}, t) \le q < \tilde{\Phi}(D_{j_1+1}, t)$ and $\tilde{\Phi}(D_{j_2}, t) < q \le \tilde{\Phi}(D_{j_2+1}, t)$. Note that a binary search, as well as more effective methods, can be used for the root finding, and thus the discrete-threshold approximation to adaptive EARL can have significant advantages over the state-of-art numerical algorithms for rank filtering, especially when operating on large time scales.

Discrete-Threshold Approximation to AARF   It is worth pointing out that the invariance of rank to a monotonic transformation allows us to define the following discrete-threshold approximation to an adaptive analog rank filter:

$$\dot{\overline{D}}_q(t) = \frac{K(t)\left\{q - \mathcal{F}_{\Delta\overline{D}}\left[\overline{D}_q(t) - \overline{x}(t)\right]\right\}}{\tau\, h_\tau(t) * \left\{K(t)\, f_{\Delta\overline{D}}\left[D_k(t) - \overline{x}(t)\right]\right\}},\tag{42}$$

where $D_k(t) = \delta D\, k(t) = \delta D\, \mathrm{nint}(\overline{D}_q(t)/\delta D),$[8] and $\delta D \ll \Delta\overline{D}$.

4.3   Example: Bimodal Analog Sensor Interface System (BASIS)

BASIS constitutes an *analog* signal processing module, initially intended to be coupled with a photon counting sensor such as a photomultiplier tube (PMT). The resulting integrated photodetection unit allows fast and sensitive measurements in a wide range of light intensities, with adaptive automatic transition from counting individual photons to the continuous (current) mode of operation. When a BASIS circuit is used as an external signal processing unit of a photosensor, its output $R_{\mathrm{out}}(t)$ is a continuous signal for both photon counting and current modes, with a magnitude proportional to the rate of incident photons. This signal can be, for example, used directly in analog or digital measuring and/or control systems, differentiated (thus producing continuous time derivative of the incident photon rate), or digitally sampled for subsequent transmission and/or storage. Thus, BASIS converts the raw output of

---

[8]The nearest integer function $\mathrm{nint}(x)$ is defined as the integer part of $x + \frac{1}{2}$, $\mathrm{nint}(x) = \lfloor x + \frac{1}{2} \rfloor$.

a photosensor to a form suitable for use in continuous action light and radiation measurements.

The functionality of the BASIS is enabled through the integration of three main components: (1) Analog Counting Systems (ACS), (2) Adaptive Analog Rank Filters (AARF), and (3) Saturation Rate Monitors (SRM), as described further. The BASIS system pro-
5 vides several significant advantages with respect to the current state-of-art signal processing of photosignals. Probably the most important advantage is that, by seamlessly merging the counting and current mode regimes of a photosensor, the output of the BASIS system has a contiguous dynamic range extended by 20-30 dB. This technical enhancement translates into important commercial advantages. For example, the extension of the maximum rate of the
10 photon counting mode of a PMT by 20 dB can be used for a tenfold increase in sensitivity or speed of detection. Since sensitivity and speed of light detecting units is often the bottleneck of many instruments, this increase will result in upgrading the class of equipment at a fraction of the normal cost of such an upgrade.

In addition, the analog implementation of the current mode regime reduces the overall
15 power consumption of the detector. These capabilities will benefit applications dealing with light intensities significantly changing in time, and where autonomous low-power operation is a must. One particular example of such an application is a high sensitivity handheld radiation detection system that could be powered with a small battery. Such a compact detector could be used by United States customs agents to search for nuclear materials entering the country.
20 <u>Principal components (modules) of BASIS</u> As shown in figure 23, the principal components (modules) of the BASIS can be identified as (I) Rank Filtering (or Baseline) Module, (II) Analog Counting Module (ACM), (III) the Saturated Rate Monitor (SRM), and (IV) Integrated Output Module. A brief description of these modules is as follows.

<u>Rank Filtering (or Baseline) Module</u> As shown in figure 23, the Baseline Module outputs the
25 rank-filtered signal $D_q(t; T)$, which is the qth quantile of the signal $x(t)$ in a moving time window of characteristic width T. The rank filtering is accomplished by means of an Adaptive Analog Rank filter (AARF) (see §4.1), or its single-point version referred to as a Single Point Analog Rank Tracker (SPART) (*ibidem*). AARFs, due to their insensitivity to outliers, are essential for stable operation of BASIS, and are used to create, maintain, and modify its

41

analog control levels (the control levels of the comparators in the Threshold Domain Filter). For example, a baseline created by an AARF operating as a *median filter* (i.e., $q = 1/2$) will not significantly change its value unless the photoelectron rate exceeds about half of the saturation rate R_{max}. On the other hand, this baseline will track the changes in the noise

5   level, providing an effective separation between noise and the photosignal.

When the photoelectron rate exceeds the saturation rate R_{max}, the output of the AARF itself will well represent the central tendency of the photosignal, and thus will be proportional to the incident photon rate. In the 'transitional' region (around R_{max}), the output of the BASIS can be constructed as a weighted sum of the outputs of AARF and ACM. Thus the

10   total output of BASIS can be constructed as a combination of the outputs of AARF, ACM, and SRM, and calibrated to be proportional to the incident photon rate.

Analog Counting Module (ACM) This module produces a continuous output, $R(t)$, equal to the rate of upward zero crossings of the difference, $x(t) - rD_q(t; T)$, in the time window, $w(t)$, given by

15
$$R(t) = w(t) * \mathcal{R}(t) = w * \left| \frac{d}{dt} \theta(x - rD_q) \right|_+ , \qquad (43)$$

where $\mathcal{R}(t)$ denotes the *instantaneous crossing rate* (Nikitin et al., 2003). The value of the parameter r generally depends on the distribution of the photosensor's noise in relation to the single photoelectron distribution of the photosensor, and can normally be found either theoretically or empirically based on the required specifications. This parameter affects the

20   ratio of the false positive (noise) and the false negative counts (missed photoelectrons) and allows us to achieve a desired compromise between robustness and selectivity. In the subsequent simulated example (see figure 25), the quantile parameter $q = 1/2$ (AARF operating as a median filter), and $r = 6$. An attractive choice for the time averaging filter $w(t)$ is a sequence of 3 RC-integrators with identical time constants $\tau = T/6$, which will provide us with

25   rate measurements corresponding to the time averaging with a rectangular moving window of width T (Nikitin et al., 2003).

The main advantage of the analog counting represented by equation (43) is a complete absence of *dead time* effects (see Nikitin et al. 2003). In addition, the baseline created by

an AARF will not be significantly affected by the photoelectron rates below approximately $(1 - q) R_{\mathrm{max}}$ (half of the photosensor saturation rate for a median filter). Thus the maximum measured rate is limited only by the single electron response of a photosensor. This is at least two orders of magnitude higher than the current state of the art photon counting systems. For

5   example, in the simulation presented in figure 25, the FWHM of the single electron response is about 1 ns, and the saturation rate of photon counting is about 3×10^8 s^{-1}. Since the signal-to-noise ratio is proportional to the square root of the rate, the 20 dB increase in the photon counting rate translates into a tenfold increase either in the sensitivity or the speed of detection.

10   <u>Saturation Rate Monitor (SRM)</u>   The SRM produces a continuous output $R_{\mathrm{max}}(t)$ equal to the rate of upward zero crossings of the difference $x(t) - D_{1/2}(t; T)$ in the time window $w(t)$,

$$R_{\mathrm{max}}(t) = w * \left| \frac{\mathrm{d}}{\mathrm{d}t} \theta(x - D_q) \right|_+ . \tag{44}$$

As was theoretically derived by Nikitin et al. (1998), R_{max} is approximately equal to the maximum rate of upward (or downward) crossings of *any* constant threshold by the pho-

15   tosensor signal $x(t)$. When the photoelectron rate λ_{PhE} of a photosensor is much smaller than R_{max}, the pileup effects are small, and the photosensor is in a photon counting mode. When $\lambda_{PhE} \gg R_{\mathrm{max}}$, the photosensor is in a current mode.

Thus monitoring R_{max} allows us to automatically handle the transition between the two modes. The horizontal gray line in panel I of figure 25 shows the measured R_{max} as a function

20   of the photoelectron rate λ_{PhE}. The measured saturation rate is also shown by the horizontal thin solid lines in the lower half of panel III of figure 25.

<u>Integrated Output Module</u>   As shown in figure 24, the output module of BASIS combines the outputs of the ACM, SRM, and AARF into a single continuous output $R_{\mathrm{out}}(t)$. The magnitude of $R_{\mathrm{out}}(t)$, for both the photon counting and the current modes, is proportional to

25   the rate of incident photons. This signal can be, for example, used directly in analog or digital measuring and/or control systems, differentiated (thus producing continuous time derivative of the incident photon rate), or digitally sampled for the subsequent transmission and/or storage. For the simulated example shown in figure 25, $R_{\mathrm{out}}(t)$ was chosen as the following

43

combination of $D_q(t;T)$, $R(t)$, and $R_{\max}(t)$:

$$R_{\text{out}}(t) = R(t) + \beta D_q(t;T)\,\mathcal{F}_{\Delta D}\left[\beta D_q(t;T) - \gamma R_{\max}(t)\right], \qquad (45)$$

where β is a calibration constant, $\Delta D = \alpha R_{\max}$, α being a small number (of order 10^{-1}), and $\gamma \sim 1/2$ is a quantile constant. The Integrated Output Module thus includes the 'transitional'

5   region between the photon counting and the current modes (shaded in gray in figure 25), currently unavailable, into a normal operational range of a photosensor, extending by $\sim$20 dB the photosensor's contiguous dynamic range.

Simulated examples of light measurements conducted by PMT with BASIS unit   Figure 25 provides a simulated example of the performance of BASIS used with a PMT. In the sim-

10 ulation, a fast PMT was used (the FWHM of the single electron response is about 1 ns), and the noise rate was chosen to be high (order of magnitude higher than the PMT saturation rate). *Panel I* of the figure shows the output of BASIS (R_{out}, thick solid black line) as a function of the photoelectron rate λ_{PhE}, along with the outputs of the Saturation Rate Monitor ($R_{\max}$, solid gray line), Rank Filtering Module ($D_{1/2}$, dashed line), and Analog Counting

15 Module (R, thin solid black line). *Panel II* shows (by gray lines) 1 μs snapshots of the PMT signal for the photoelectron rates much smaller (left), approximately equal (middle), and much higher (right) than the saturation rate of the PMT. This panel also shows (by the black lines) the respective outputs of the Rank Filtering Module $D_{1/2}(t)$, and the baseline levels $rD_{1/2}(t)$ used in the Analog Counting Module ($r = 6$ in the simulation). The instantaneous crossing

20 rates $\mathcal{R}(t)$ are also shown (top), and the time constant T of AARF is indicated in the lower left corner of the panel. *Panel III* illustrates the relation between the noise and photosignal used in the simulation by depicting the accumulated amplitude and counting distributions of the PMT signal. These distributions are shown for three chosen photoelectron rates λ_{PhE}, in their relation to the outputs of the Rank Filtering Module ($D_{1/2}$) and the Saturation Rate Monitor

25 ($R_{\max}$). The resolution ΔD of the acquisition system used for measuring the distributions is indicated in the panel.

Figure 26 provides a simulated example of a modification of BASIS designed for detection of fast changes in a light level. The light signal corresponding to this model can be, for

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example, an intensity modulated light signal passing through a fiber, or fluorescence of dye
excited by an action potential wave propagating through a biological tissue. The gray line in
the lower panel of the figure shows the time-varying light signal (square pulses). The higher
light level corresponds to the photoelectron rate of about 2×10^9 photoelectrons per second.

5   The width (FWHM) of the single electron response of the photosensor is about 1 ns, and the
resulting photosensor electrical signal $x(t)$ is shown by the gray line in the middle panel.

As can be seen in the figure, the low signal-to-noise ratio makes fast and accurate deduction
of the underlying light signal difficult. The circuit shown at the top of figure 26, however,
allows reliable timing of the onsets and offsets of the light pulses with better than 10 ns

10   accuracy. The output of the circuit $D_{q_f}(t; T_f)$ is shown by the black line in the lower panel
of the figure. In the example, the quantile parameters of the rank filters are $q_b = 1/4$ and
$q_f = 3/4$, and the baseline factor is $r = 1.5$. The parameter r allows us to adjust the circuit
for optimal performance based on the difference between the low and high light levels.

5   Generation of monoenergetic Poissonian pulse trains

As another illustration of the current invention, consider a technique and a circuit for genera-
tion of monoenergetic Poissonian pulse trains with adjustable rate and amplitude. Generators
of such pulse trains can be used, for example, in testbench development and hardware proto-
5   typing of instrumentation for nuclear radiation measurements.

Idealized model of a Poisson pulse train generator   An idealized process producing a monoen-
ergetic Poissonian pulse train can be implemented as schematically shown in figure 27. Con-
sider a stationary random pulse train $\sum_i x_i \, \delta(t - t_i)$, where x_i are the amplitudes of the pulses
with the arrival times t_i. This pulse train is filtered by a linear time filter with a *continuous*
10   impulse response $w_{\Delta\tau}(t)$, where $\Delta\tau$ is the characteristic response time of the filter. An exam-
ple of such a response would be the bipolar pulse $w_{\Delta\tau}(t) = \left[t/\Delta\tau^2 - t^2/(2\Delta\tau^3) \right] e^{-t/\Delta\tau} \theta(t)$,
where θ is the Heaviside unit step function. The output $x(t)$ of the linear time filter can be
written as

$$x(t) = \sum_i x_i \, w_{\Delta\tau}(t - t_i), \tag{46}$$

15   and is a continuous signal. The *instantaneous* rate of upward crossings Nikitin et al. (2003)
of a threshold D by this signal can be written as

$$\mathcal{R}(D, t) = \left| \frac{\mathrm{d}}{\mathrm{d}t} \theta \left[x(t) - D \right] \right|_+ = \sum_j \delta(t - t_j), \tag{47}$$

where t_j are the instances of the crossings (that is, $x(t_j) = D$ and $\dot{x}(t_j) > 0$). As was
discussed in Nikitin (1998), the pulse train given by equation (47) is an approximately Pois-
20   sonian train affected by a non-extended dead time of order $R_{\max}^{-1}$. Thus, when the average
rate $R(D) = \langle \mathcal{R}(D, t) \rangle_T$ is much smaller than the saturation rate $R_{\max}$, $\mathcal{R}(D, t)$ will provide
a good approximation for a monoenergetic Poissonian pulse train of the average rate $R(D)$.

When either $M_1 = 0$ or $W_{10} = 0$, then, as was shown in Nikitin et al. (1998), the *average*
rate of the upward crossings of a threshold D by the signal $x(t)$ can be expressed as

$$R(D) = R_{\max} \exp\left[-\frac{1}{2}\left(\frac{D}{\sigma}\right)^2\right] , \tag{48}$$

and thus the rate of the generated pulse train can be adjusted by an appropriate choice of the threshold value D.

<u>Practical implementation of a Poisson pulse train generator</u> The idealized process described above is not well suited for a practical generation of a Poissonian pulse train, since, as can be seen from equation (48), at high values of the threshold D the rate of the generated train is highly sensitive to the changes in D. To reduce this sensitivity, one can pass the signal $x(t)$ through a nonlinear amplifier, e.g., an antilogarithmic amplifier as shown in figure 27, thus transforming $x(t)$ into the signal $y(t) = \exp\left[x(t)/\sigma\right]$. Then the average rate of the upward crossings of a threshold D by the signal $y(t)$ can be written as

$$R(D) = R_{\max} \exp\left\{-\frac{1}{2}\left[\ln\left(\frac{D}{\sigma}\right)\right]^2\right\} , \tag{49}$$

which is much less sensitive to the relative errors in D.

Figure 28 illustrates a simulated performance of an idealized monoenergetic Poisson pulse generator shown in figure 27. The upper panel of figure 28 shows the output pulse rates as a function of threshold, and the lower panels show the distributions of the pulses' interarrival times for the generator set at two different threshold values. In the figure, the black solid lines show the theoretical curves, and the gray solid lines show the respective results of the simulations.

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ARTICLES OF MANUFACTURE

Various embodiments of the invention may include hardware, firmware, and software embodiments, that is, may be wholly constructed with hardware components, programmed into firmware, or be implemented in the form of a computer program code.

5   Still further, the invention disclosed herein may take the form of an article of manufacture. For example, such an article of manufacture can be a computer-usable medium containing a computer-readable code which causes a computer to execute the inventive method.

REFERENCES

B. C. Arnold, N. Balakrishnan, and H. N. Nagaraja. *A First Course in Order Statistics.* John Wiley & Sons, Inc., 1992.

J. S. Bendat. *Nonlinear system techniques and applications.* Wiley, New York, 1998.

5   N. Bleistein and R. A. Handelsman. *Asymptotic Expansions of Integrals.* Dover, New York, 1986.

A. C. Bovik, T. S. Huang, and Jr. D. C. Munson. A generalization of median filtering using linear combinations of order statistics. *IEEE Trans. Acoust., Speech, Signal Processing,* ASSP-31:1342–1350, 1983.

10   A. S. Davydov. *Quantum Mechanics.* International Series in Natural Philosophy. Pergamon Press, 2nd edition, 1988. Second Russian Edition published by Nauka, Moscow, 1973.

P. J. S. G. Ferreira. Sorting continuous-time signals and the analog median filter. *IEEE Signal Processing Letters,* 7(10):281–283, 2000.

P. J. S. G. Ferreira. Sorting continuous-time signals: Analog median and median-type

15   filters. *IEEE Trans. Signal Processing,* 49(11): 2734–2744, Nov. 2001.

V. Kim and L. Yaroslavsky. Rank algorithms for picture processing. *Computer Vision, Graphics and Image Processing,* 35:234–258, 1986.

P. Kinget and M. Steyaert. *Analog VLSI integration of massive parallel processing systems.* Kluwer, 1997.

20   C. L. Lee and C.-W. Jen. Binary partition algorithms and VLSI architectures for median and rank order filtering. *IEEE Transactions on Signal Processing,* 41(9):2937–2942, 1993.

C. Mead. *Analog VLSI and neural systems.* Addison-Wesley, 1989.

N. R. Murthy and M. N. S. Swamy. On the VLSI implementation of real-time order statistic filters. *IEEE Transactions on Signal Processing,* 40(5):1241–1252, 1992.

25   A. V. Nikitin. *Pulse Pileup Effects in Counting Detectors.* PhD thesis, University of Kansas, Lawrence, 1998.

A. V. Nikitin and R. L. Davidchack. Method and apparatus for analysis of variables.

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Geneva: World Intellectual Property Organization, International Publication Number WO 03/025512, 2003.

A. V. Nikitin and R. L. Davidchack. Signal analysis through analog representation. *Proc. R. Soc. Lond. A*, 459(2033):1171–1192, 2003.

5 A. V. Nikitin, R. L. Davidchack, and T. P. Armstrong. The effect of pulse pile-up on threshold crossing rates in a system with a known impulse response. *Nucl. Instr. & Meth.*, A411:159–171, 1998.

A. V. Nikitin, R. L. Davidchack, and T. P. Armstrong. Analog multivariate counting analyzers. *Nucl. Instr. & Meth.*, A496(2-3):465–480, 2003.

10 E. Ochoa, J. P. Allebach, and D. W. Sweeney. Optical median filtering using threshold decomposition. *Applied Optics*, 26(2):252–260, Jan. 1987.

I. E. Opris. *Analog Rank Extractors and Sorting Networks*. Ph.D. Thesis, Stanford University, CA, 1996.

I. Osorio, M. G. Frei, and S. B. Wilkinson. Real-time automated detection and quantitative analysis of seizures and short-term prediction of clinical onset. *Epilepsia*, 39(6):615–627, 1998.

15 S. Paul and K. Hüper. Analog rank filtering. *IEEE Trans. Circuits Syst.—I*, 40(7):469–476, Jul. 1993.

K. Urahama and T. Nagao. Direct analog rank filtering. *IEEE Trans. Circuits Syst.—I*, 42(7):385–388, Jul. 1995.

20 S. Vlassis, K. Doris, S. Siskos, and I. Pitas. Analog implementation of erosion/dilation, median and order statistics filters. *Pattern Recognition*, 33(6):1023–1032, 2000.

Regarding the invention being thus described, it will be obvious that the same may be varied in many ways. Such variations are not to be regarded as a departure from the spirit and scope of the invention, and all such modifications as would be obvious to one skilled in the art are intended to be included within the scope of the following claims.

We claim:

1. Method for processing an input signal consisting of a Threshold Domain Filtering step utilizing a threshold domain and at least one of the following additional steps:

   (a) Multimodal Pulse Shaping;

   (b) Analog Rank Filtering; and

   (c) Analog Counting.

2. Method for signal processing of claim 1 wherein said Threshold Domain Filtering step is used for extraction of one or more features of interest from said input signal to produce one or more output signals.

3. Method for signal processing of claim 1 wherein said threshold domain is defined by a Control Level Signal consisting of one or more components.

4. Method for signal processing of claim 1 wherein said Threshold Domain Filtering step outputs a first output value when said input signal is within said threshold domain and a second output value when said input signal is outside of said threshold domain.

5. Method for signal processing of claim 1 wherein said Threshold Domain Filtering step outputs a first output value when said input signal is within said threshold domain, a second output value when said input signal is outside of said threshold domain, and one or more values when said input signal is inside a finite interval at a boundary of said threshold domain.

6. Method for signal processing of claim 1 wherein at least one of said additional steps is Multimodal Pulse Shaping and wherein said input signal has one or more components and wherein said Multimodal Pulse Shaping transforms at least one of said components of said input signal into a plurality of signal components where at least one said component represents a derivative of another said component.

7. Method for signal processing of claim 6 wherein said derivative is a time-derivative.

8. Method for signal processing of claim 6 wherein said plurality of signal components is representative of one or more features of interest of said input signal.

9. Method for signal processing of claim 3 wherein one or more components of said Control Level Signal are outputs of a step of Analog Rank Filtering.

10. Method for signal processing of claim 3 wherein one or more components of said Control Level Signal are results of a step of Analog Rank Filtering of one or more output components of a step of Multimodal Pulse Shaping of said input signal.

11. Method for signal processing of claim 1 wherein at least one of said additional steps is Analog Counting, wherein said Threshold Domain Filtering step has an output with at least one component and where said Analog Counting comprises the steps of:

   (a) time-differentiating said component to produce a time-derivative, and

   (b) rectifying said time-derivative to produce an instantaneous count rate.

12. Method for signal processing of claim 11 wherein said Analog Counting further comprises the step of:

   (a) time-integrating said instantaneous count rate to produce a count rate in a moving time window.

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13. Method for Adaptive Analog Rank Filtering operable to transform an input scalar variable into an output Rank Filtered Variable comprising steps of:

(a) forming a plurality of outputs of delayed comparators by passing said input variable and a plurality of feedbacks of Offset Rank Filtered Variables through a respective plurality of said delayed comparators;

(b) forming a first difference which is a weighted difference of said outputs of the delayed comparators;

(c) forming a Density Function by time averaging said first difference;

(d) forming a second difference which is a weighted difference of said feedbacks of the Offset Rank Filtered Variables;

(e) forming a plurality of differences between the outputs of the delayed comparators and the respective Offset Quantile Parameters of said Offset Rank Filtered Variables;

(f) forming a plurality of time derivatives of said Offset Rank Filtered Variables by multiplying each of said plurality of differences by a ratio of said second difference and said Density Function;

(g) producing the plurality of the Offset Rank Filtered Variables by time-integrating said plurality of time derivatives; and

(h) producing said Rank Filtered Variable as a weighted average of said Offset Rank Filtered Variables.

14. Method for Adaptive Analog Rank Selecting operable to transform a plurality of input variables into an output Rank Selected Variable comprising steps of:

   (a) forming a plurality of outputs of averaging comparators by passing said plurality of input variables and each component of a plurality of feedbacks of Offset Rank Selected Variables through a respective plurality of said averaging comparators;

   (b) forming a first difference which is a weighted difference of said outputs of the averaging comparators;

   (c) forming a Density Function by time averaging said first difference;

   (d) forming a second difference which is a weighted difference of said feedbacks of the Offset Rank Selected Variables;

   (e) forming a plurality of differences between the outputs of the averaging comparators and the respective Offset Quantile Parameters of said Offset Rank Selected Variables;

   (f) forming a plurality of time derivatives of said Offset Rank Selected Variables by multiplying each of said plurality of differences by a ratio of said second difference and said Density Function;

   (g) producing the plurality of the Offset Rank Selected Variables by time-integrating said plurality of the time derivatives; and

   (h) producing said Rank Selected Variable as a weighted average of said Offset Rank Selected Variables.

15. Method for signal processing of claim 3 wherein one or more components of said Control Level Signal are linear combinations of outputs of a step of Adaptive Analog Rank Filtering of one or more components of said input signal.

16. Method for signal processing of claim 3 wherein one or more components of said Control Level Signal are linear combinations of outputs of a step of Adaptive Analog Rank Filtering of one or more output components of a step of Multimodal Pulse Shaping of said input signal.

17. Method for signal processing of claim 3 wherein one or more components of said Control Level Signal are linear combinations of outputs of a step of Adaptive Analog Rank Selecting of a plurality of components of said input signal.

18. Method for signal processing of claim 3 wherein one or more components of said Control Level Signal are linear combinations of outputs of a step of Adaptive Analog Rank Selecting of a plurality of output components of a step of Multimodal Pulse Shaping of said input signal.